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AN EQUIVALENT STRESS METHOD FOR CONSIDERING LOCAL BUCKLING IN BEAM FINITE ELEMENTS IN THE FIRE SITUATION

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ABSTRACT

The use of slender steel sections has increased in recent years because they provide excellent strength to weight ratio. Yet, a major issue with slender sections is local buckling in compression zones. Several researchers have proposed design methods at elevated temperatures based on the effective width approach to calculate the capacity of the plates that compose these steel members, but this approach is not easily compatible with the implementation and use in Bernoulli beam elements. Another approach is the development of a stress based model, i.e. an "effective" constitutive law of steel. This approach was proposed previously by Liege University researchers for slender steel members exposed to high temperatures, and implemented within the framework of fiber type Bernoulli beam elements; however it was giving overly conservative results. This paper presents an improved temperature-dependent constitutive model for steel that accounts for local instabilities using the stress based method. The improved model is derived from refined plate analysis methodology and implemented in the SAFIR finite element analysis software. Validation shows good agreement against experimental and shell element analysis results.

Keywords: Local buckling, slender cross-sections, fire, beam finite elements, stress based method

1 INTRODUCTION

The use of slender steel sections has increased in recent years because they provide excellent strength to weight ratio; this trend has also been favoured by the development of higher steel grades. Yet, a major issue with slender sections is local buckling that may occur in compression zones: in the flange under compression for elements in bending and essentially in the web for elements in compression. In very deep sections, shear can also trigger local buckling in the web if it is too slender. Local buckling can also have a significant influence on the behaviour of steel members in the fire situation. Detailed information regarding the local buckling of steel members exposed to fire is presented in [1].

Furthermore, past fire accidents have demonstrated local buckling failures in structural members with slender cross sections, like in WTC 5 [2] and Broadgate fire [3].

To take local instabilities into account, several design methods have been proposed by researchers relying on the effective width approach and based on numerical models of isolated plates [4], [5], [6] or analytical methods [7], but this approach is not easily compatible with the implementation and use in Bernoulli beam elements. Beyond the complexities associated with estimating the effective width (which formally depends on the stress distribution), the development of thermal strains in the fire situation may lead to reversal from tension to compression, and vice versa, in

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different parts of the cross section, rendering the effective width approach particularly difficult to implement. Therefore another approach was proposed by Liege University researchers as the development of a stress based model, i.e. an “effective” constitutive law of steel [8]. This effective stress based approach for slender steel members exposed to high temperatures was implemented within the framework of fiber type Bernoulli beam finite elements. However the implemented model was giving overly conservative results.
This paper presents an improved temperature-dependent constitutive model for steel that accounts for local instabilities using the stress based method. Based on a refined methodology and revisited assumptions [9], [10], [11], improved buckling reduction factor vs plate slenderness relationships have been derived and, based on these relationships, a novel equivalent stress-strain-temperature relationship has been derived. The model is intended to be used in nonlinear numerical analysis with fiber type beam finite elements aiming at calculation of the fire resistance of thin-walled steel elements. The model can be easily implemented into any finite element software which include this type of FE.

2 PROPOSED MODEL

2.1 Model development

The effective law is derived with the same objective as the effective width: the compressive capacity obtained with the effective law in the full section should be equal to the capacity of the slender plate with the real material under local buckling. Because local buckling develops only in compression, the real stress-strain relationship needs to be modified only in compression and remains untouched in tension. It thus leads to an effective law that is non-symmetrical with respect to compression-tension. The tangent modulus at the origin of the law is not modified (because low compression stresses do not produce local instabilities), but the development of local instabilities is reflected by a reduction of the limit of proportionality and of the effective yield strength. The effective stress-strain relationship in compression depends on the slenderness, on the boundary conditions of the plates and on the temperature.
The model development is based on parametric finite element analyses of isolated plates in pure compression. Steady-state compression tests are performed on the plates at different (constant) high temperatures. The analyses are performed for three sides simply supported (flange equivalent) and four sides simply supported plates (web equivalent), for steel grades from S235 and up to S460 and for temperatures from 20°C to 900°C. The following assumptions are adopted: imperfection amplitude equal to 1/200 of the plate width b [9]; sinusoidal shape of imperfections with m half-waves (m = 1 for webs and m = 1 and 4 for flanges) [10], [11], and plate length ratio a/b equal to 1 for web and 5 for flange [10]. The software SAFIR® [12] is used for the finite element analyses. Typical analysis results are presented in *Fig. 1* that presents the critical stress, defined as the critical buckling load divided by the cross section area, as a function of the slenderness ratio for different temperatures.
The results can be rewritten in terms of buckling reduction factor, defined as the critical stress divided with the yield stress, as a function of the non-dimensional plate slenderness. The non-dimensional plate slenderness is defined according to *Eq. 1* which is taken from EN 1993-1-5 [13]:

$$\lambda_{p,\theta} = \frac{b/t}{28.4\varepsilon\sqrt{k_\sigma}} \qquad (1)$$

where k_σ is a factor considering the applied boundary conditions defined in EN 1993-1-5 [13] and ε has been taken for elevated temperatures from the equation:

$$\varepsilon = \sqrt{\frac{k_{E,\theta}}{k_{y,\theta}}}\sqrt{\frac{235}{f_y}} \qquad (2)$$

where $k_{E,\theta}$ and $k_{y,\theta}$ are the reduction factors of Young’s modulus and yield strength respectively for temperature θ [14].

The results are plotted in *Fig. 2.* As it can be seen, all analysis results are lying within a narrow band except for the curves at temperatures of 20 °C and 200°C. Therefore, it seems reasonable to adopt a single buckling reduction factor curve for the temperatures in the range of 300°C to 900°C.

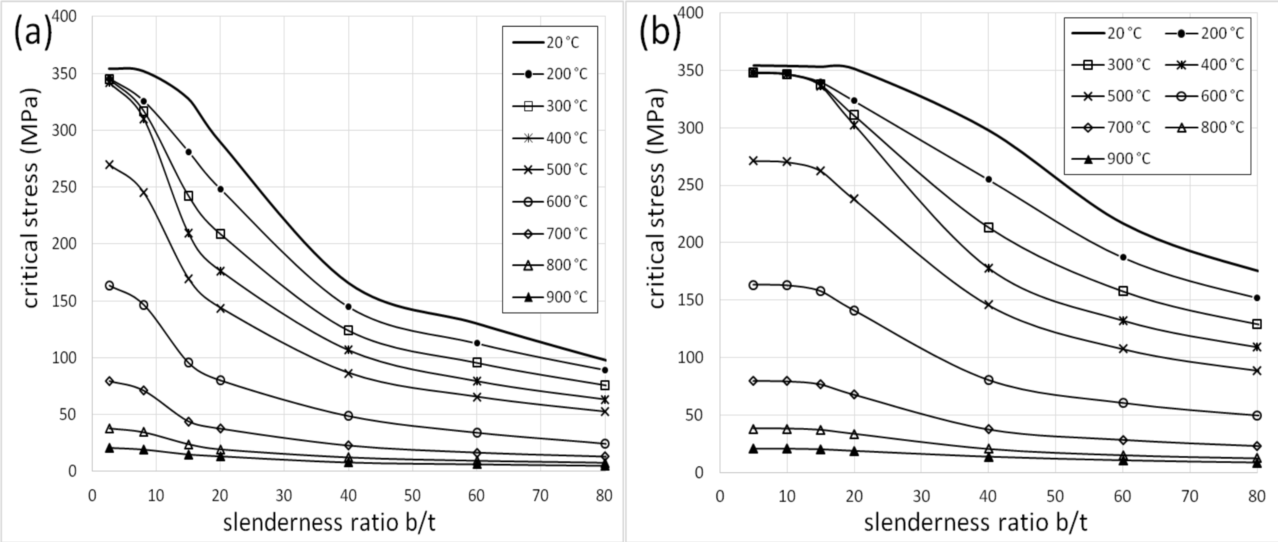

Fig. 1. Isolate plate (S355) analysis results (a) three sides simply supported (flange equivalent); (b) four sides simply supported (web equivalent)

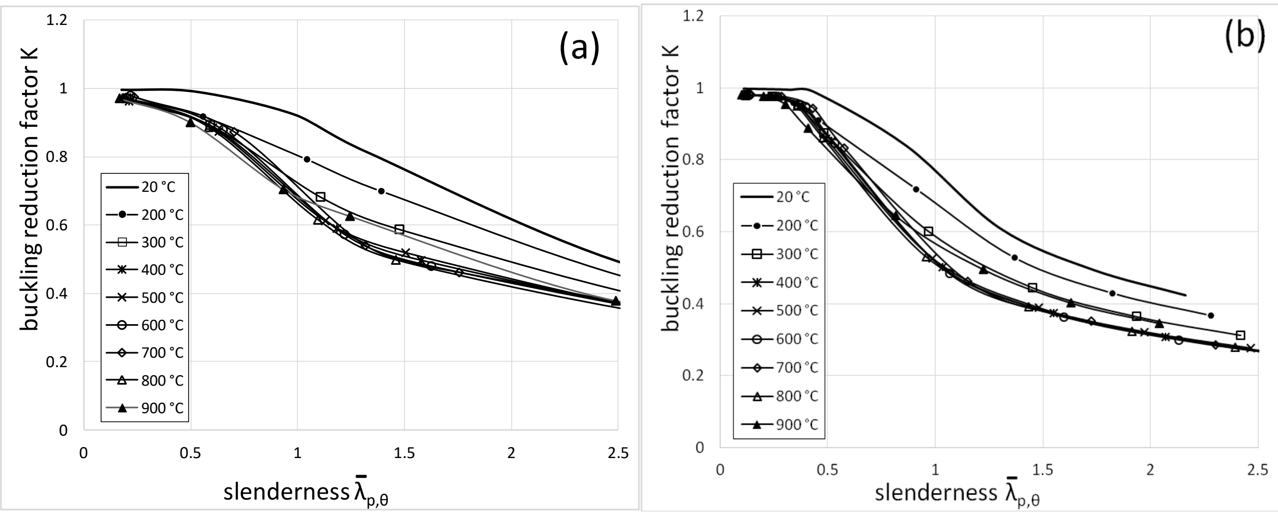

Fig. 2. Buckling reduction factor vs slenderness for S355 plates (a) three sides simply supported (flange equivalent); (b) four sides simply supported (web equivalent)

2.2 Proposed plate buckling model

Eq. 3 is proposed to model the buckling curves of *Fig. 2*:

$$k_\theta = \frac{1}{\alpha \cdot \lambda_{p,\theta}^2 + \beta \cdot \lambda_{p,\theta} + \gamma} \tag{3}$$

where k_θ is the buckling reduction factor at temperature θ and α, β and γ are model parameters.

A statistical analysis based on the least square method is conducted on the results of the isolated plate analyses to establish the model parameters (see *Fig. 3*). The obtained values are presented in *Table 1* for the parameters α, β and γ, estimated at 20°C, 200°C, and 300°C-900°C, for flange and web plates (six sets of parameters). It was chosen to keep the proposed model for buckling reduction factor as a function of plate slenderness independent of the steel grade to reduce the number of parameters and hence simplify its application; this approximation has only a slight influence on the model accuracy.

Table 1. Proposed model parameters

Type of plate	Temperature (^oC)	α	β	γ
Flange	20	-0.19800	1.375	-0.0368
	200	-0.10000	1.000	0.6350
	300 and higher	-0.05500	1.130	0.6200
Web	20	-0.00066	0.446	0.9000
	200	0.04860	0.723	0.7400
	300 and higher	-0.03100	1.347	0.5300

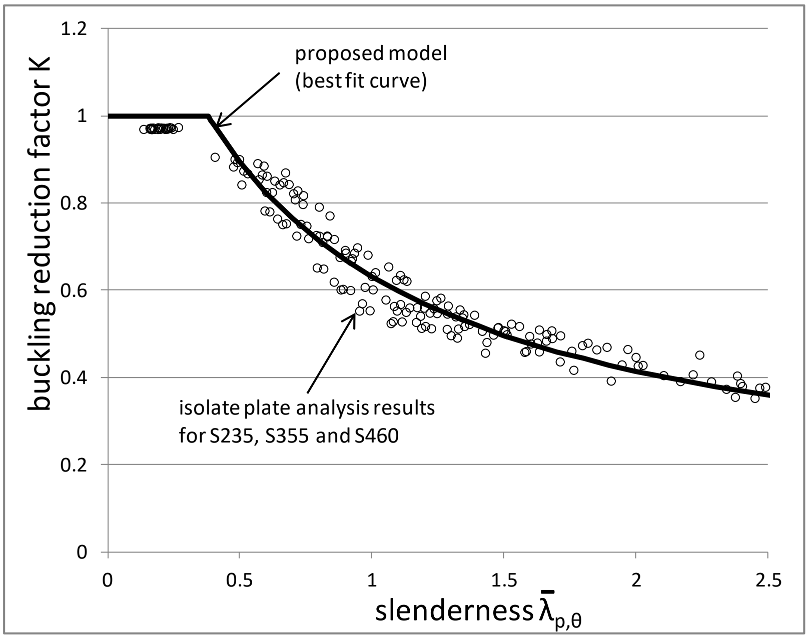

Fig. 3. FEA results and proposed model curve for the isolated four sides simply supported plate for grades S235, S355 and S460 and temperatures equal or higher than 300°C.

2.3 Stress-strain-temperature relationship

A modified stress-strain-temperature relationship for steel is derived based on EN1993-1-2 [14] and the proposed plate buckling model. The tension part of the EN1993-1-2 material law remains unchanged. In compression, the proportional limit and the yield strength are reduced by the factor k_θ (*Eq. 3*). The strain corresponding to the compressive yield strength is also reduced by the factor k_θ. As a result, the proposed material law is asymmetric in tension and compression, and a different law is used for the flanges and for the web as different factors k_θ apply (*Fig. 4*).

The proposed model was implemented in the software SAFIR [12] where it is used in every point of integration in the beam elements. This means that, for example, the strain varies in the web of a section under bending and, as a consequence, also the factor k_θ.

3 VALIDATION OF PROPOSED MODEL

3.1 Validation against fire test results

In order to validate the proposed model, experimental results collated from two different sources [15], [16] were used. The fire tests No 3 and No 5 from [15] were simulated with beam finite elements. These tests performed in Liege University Fire Labs during FIDESC4 Research Program. The fire test No 3 specimen had cross section 150x5(flanges) and 450x4(web) and length 2.7 m. The applied load was 122.4 kN. Load eccentricity of 4 mm applied in the weak direction to avoid global buckling failure due to global imperfection. The amplitude of local imperfections was 2.6 mm at the web and 4.9 mm at the flange. The fire test No 5 specimen had cross section

150x5(flanges) and 360x4(web) and length 2.7 m. The applied load was 231 kN. Load eccentricity of 71 mm applied in the strong direction at the bottom and at the top. The amplitude of local imperfections was 6.8 mm at the web and 2.3 mm at the flange. The load eccentricity of test No 5 has not been considered for the simulation for the reason explained in Section 4. The comparison between experimental results and beam element analysis results is presented in *Fig. 5* and shows good agreement with the tests. A clear improvement from the original model proposed in [8] is also visible.

The simulation of the tests from [16] considered only the heated part of the column and the comparison between experimental results and beam element analysis results is presented in Table 2, showing good agreement.

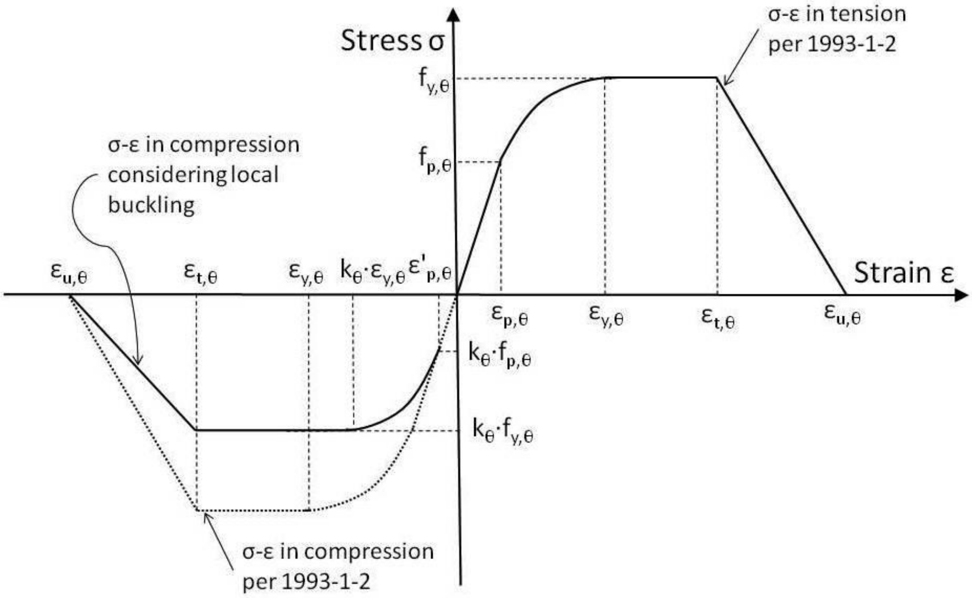

Fig. 4. Stress-strain relationship per EN1993-1-2[14] compared with the proposed modified relationship.

Table 2. Validation against experimental results form [17]

Cross section (mm)	Ambient temperature yield stress (MPa)	Temperature (^{o}C)	Buckling resistance (kN)[17]	Buckling resistance (kN) Beam element - proposed model
250x250x6x8	306.4(flange), 321.9(web)	450	800	791
316x200x6x8	306.4(flange), 321.9(web)	450	750	785
250x220x8x8	538.1	450	1500	1378
336x160x8x8	538.1	450	1400	1383
250x250x6x8	306.4(flange), 321.9(web)	650	240	269
316x200x6x8	306.4(flange), 321.9(web)	650	265	292
250x220x8x8	538.1	650	400	426
336x160x8x8	538.1	650	360	388

3.2 Validation against shell element numerical analysis results

In order to validate the proposed model against shell element analysis results, SAFIR was used to model columns under pure compression and beams under pure bending (laterally restrained to avoid lateral torsional buckling). The shell element analysis results compared with the proposed model beam element analysis results are presented in *Fig. 6* and show good agreement.

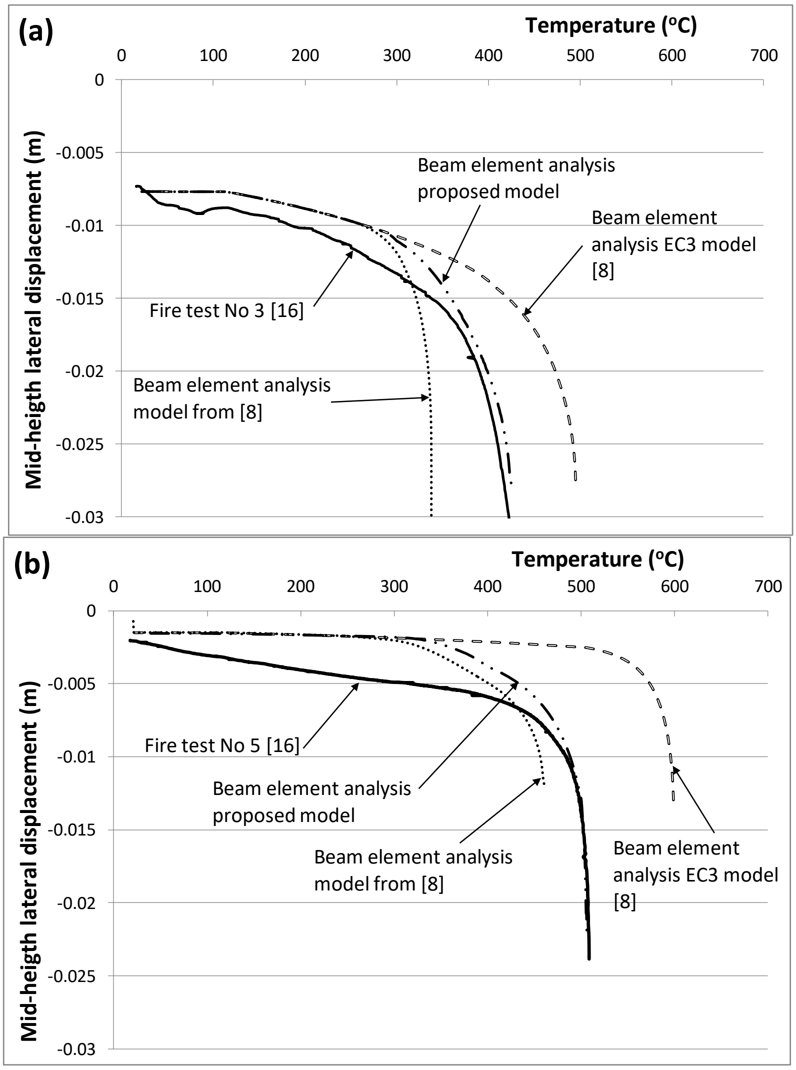

Fig. 5. Model validation against fire test results a) Test No 3[16]; b) Test No 5[16]

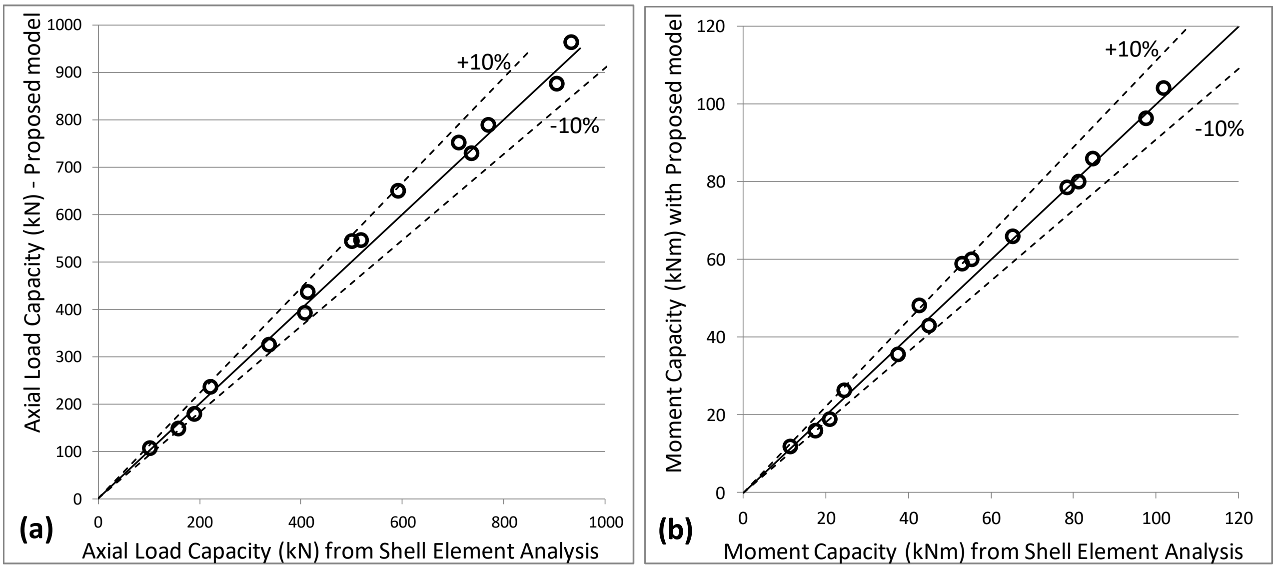

Fig. 6 Beam element analysis (proposed model) vs shell element analysis results for 150x5(flange) and 380x4(web) and temperatures from 300 °C and up to 700 °C for S235, S355 and S460 (a) Pure compression and (b) Moment capacity.

4 LIMITATIONS AND FURTHER DEVELOPMENT

The proposed model has been developed based on the assumption that a uniform displacement applies on the plate's edge [9], [10], [11]. This is accurate for columns in pure compression and it is giving good results in pure bending because of the limited bending capacity of the web. When the eccentricity is large (ψ<<1 in *Fig. 7*) the uniform displacement assumption is conservative, as it limits the compressive stress capacity to the critical uniform stress. Ongoing research aims at incorporating this effect to enhance the proposed model by considering the ψ factor. The enhanced model will be validated against fire tests which are under final preparation at the fire lab of Liège University and which will be designed to include large eccentricities to inform the model development.

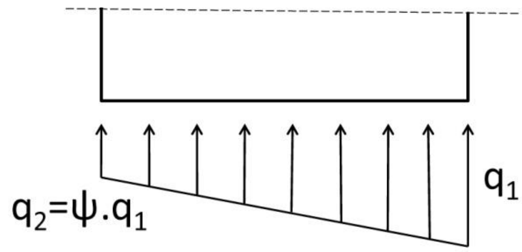

Fig. 7. Non-uniform stress distribution and definition of the parameter ψ

5 CONCLUSIONS

A model based on an equivalent stress method has been proposed as an efficient way to consider local buckling in steel members exposed to fire. The proposed stress-strain-temperature relationship is asymmetric and is modified in compression only, by reducing the proportional limit, the yield stress and the strain at yield stress. The reduction of these parameters depends on the plate's boundary conditions, slenderness and temperature.

The proposed stress-strain-temperature relationship has been implemented in the software SAFIR and validated against experimental and shell element analysis results, showing good agreement over a range of profile dimensions, temperatures and steel grades.

The proposed model is suitable for use with fibre type beam finite elements. Such a model can save enormous computational time compared with traditional shell models for thin-plate members. For example, a typical column modelled with the equivalent stress method and beam FE may have 300 degrees of freedom and a computational time shorter than 72 seconds for 350 time steps (with use of a Core i5 computer), whereas the same column modelled with shell elements would have 47750 degrees of freedom and a computational time of 847 seconds for the same time steps. It must be noted that the computational time is increased enormously when the analysis considers a larger number of structural elements.

The model is still giving conservative results for large compressive load eccentricities. For this reason an enhanced model is under development. Further extension of the proposed equivalent stress method is its modification for high and very high strength steel grades.

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REFERENCES

1. Chrysanthos Maraveas, Jean-Marc Franssen (2018). *Local buckling of steel members under fire conditions: A review,* Fire Technology (under review)
2. Therese McAllister, (2008). Federal building and fire safety investigation of the World Trade Center *disaster: structural fire response and probable collapse sequence of World Trade Center building 7*. Gaitherburg. MD, National Institute of Standards and Technology [NIST NCSTAR 1-9].
3. Yong Wang, (2002). *Steel and composite structures: behaviour and design for fire safety*, CRC Press, London.
4. Carlos Couto, C., Paulo Vila Real, Nuno Lopes, Bin Zhao (2014). *Effective width method to account for the local buckling of steel thin plates at elevated temperatures*, Thin-Walled Structures, 84, p. 134-149.
5. Carlos Couto, C., Paulo Vila Real, Nuno Lopes, Bin Zhao (2015). *Resistance of steel cross-sections with local buckling at elevated temperatures*, Constructional Steel Research, 109, p. 101-104.
6. Spencer E. Quiel, Maria E.M. Garlock, (2010). *Calculating the buckling strength of steel plates exposed to fire*, Thin-Walled Structures, 48, p. 684-695.
7. Markus Knobloch, Mario Fontana, (2006). *Strain-based approach to local buckling of steel sections subjected to fire*. Constructional Steel Research, 62, p. 44-67.
8. Jean-Marc Franssen, Baptiste Cowez, Thomas Gernay (2014). *Effective stress method to be used in beam finite elements to take local instabilities into account*. Fire Safety Science 11, 544-557. 10.3801/IAFSS.FSS.11-544.
9. Chrysanthos Maraveas, Thomas Gernay, Jean-Marc Franssen (2017) *Amplitude of local imperfections for the analysis of thin-walled steel members at elevated temperatures*, Applications of Structural Fire Engineering (ASFE'17), Manchester, UK.
10. Chrysanthos Maraveas, Thomas Gernay, Jean-Marc Franssen (2017) *Buckling of steel plates at elevated temperatures: Theory of perfect plates vs Finite Element Analysis,* 2nd International Conference on Structural Safety Under Fire and Blast Loading – CONFAB, London, UK.
11. Chrysanthos Maraveas, Thomas Gernay, Jean-Marc Franssen (2017) *Thin-walled steel members at elevated temperatures considering local imperfections: numerical simulation of isolated plates*, 9th National Conference of Steel Structures, Larisa, Greece.
12. Jean-Marc Franssen, Thomas Gernay (2017). *Modeling structures in fire with SAFIR®: Theoretical background and capabilities*, Journal of Structural Fire Engineering, 8(3):300-323. 10.1108/JSFE-07-2016-0010.
13. EN 1993-1-5 (2006). *Eurocode 3 — design of steel structures — part 1-5: plated structural elements*. Brussels: European Committee for Standardisation.
14. EN 1993-1-2 (2005). *Eurocode 3: design of steel structures — part 1–2: general rules —structural fire design*. Brussels: European Committee for Standardisation.
15. FIDESC4 (2015), *Fire design of steel members with welded or hot-rolled class 4 cross-sections*, Final report, Research program of the Research Fund for Coal and Steel.
16. Weiyong Wang, Venkatesh Kodur, Xingcai Yang, Guoqiang Li (2014) *Experimental study on local buckling of axially compressed steel stub columns at elevated temperatures*, Thin-Walled Structures, 82, 33-45.