



# Reliability based determination of material partial safety factors for cast iron beams in jack arched construction exposed to standard and natural fires

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## ABSTRACT

Cast iron beams were extensively used in many 19th century structures, especially in fireproof flooring systems (such as jack arch). Many such structures are still in use today and it is important that they fulfil the current requirements of fire resistance when there is a change of use. These structures are out of scope of modern design codes and old design codes do not provide guidance for fire resistance design. Furthermore, cast iron is a brittle material weak in tension, and there are many uncertainties in its mechanical properties at ambient and elevated temperatures due to material flaws. It is necessary to quantify the probability of structural failure and to introduce safety factors to reduce the probability of structural failure in fire to an acceptable level. This paper presents the results of a detailed study whose purpose is to derive appropriate safety factors to achieve different levels of reliability, for fire safety design of cast iron beams. In this study, a fibre analysis method has been used to calculate the moment capacity of four different types of cast iron cross section. Using randomized stress-strain-temperature relationships, based on variability of the different governing parameters (under tension: maximum stress, 0.2% proof stress, corresponding strains at maximum stress (strength) and failure; under compression: Young's modulus, proportional limit, 0.2% proof stress and the maximum stress), the probability distribution of moment capacity has been calculated. Based on the criterion of cast iron beam failure not exceeding probabilities of  $10^{-1}$ ,  $10^{-2}$  and  $10^{-3}$ , material safety factors of 1.5, 3.0 and 5.0 respectively have been obtained.

## 1. Introduction

Many 19th century historic buildings in the UK, Central and Western Europe as well as the US were built with cast iron structural elements, as main loadbearing columns and beams, especially during the period of 1820–1850 [1]. Cast iron beams are typically partially fire protected using various types of thermal insulation systems [2–4], with the jack arch floor, as illustrated in Fig. 1, being the most widely applied. Because of limited use of cast iron structures in modern construction, there has been very limited research on cast iron structures, at ambient temperature and in fire.

Cast iron structural beams exhibit different behavior from that of modern steel beams. When cast iron beams are used as part of the jack arch construction, the temperature distribution in the cast iron cross-section is severely non-uniform. Also, the stress-strain curve of cast iron does not possess the same degree of plastic behavior of steel,

which makes analyzing cast iron beams using the plastic analysis method problematic. Furthermore, cast iron behaves differently under tension and compression.

Based on extensive assessments of thermal and mechanical properties of cast iron and associated insulation materials at ambient and elevated temperatures [5–7], and new experimental data [8], the authors have proposed thermal properties for the relevant thermal insulation materials, and thermal and mechanical properties for cast iron, including the thermal expansion coefficient and stress-strain-temperature relationships [8]. More recently, the authors have developed a simplified method to calculate the moment capacity of jack arch beam cross-section at elevated temperatures [9]. The fire resistance of this type of flooring systems [7] is very sensitive to variations in the mechanical properties of cast iron at elevated temperatures. Because of large variability in these properties, there is a need to develop material safety factors for fire safety design of cast-iron structures. This is the aim of this paper.

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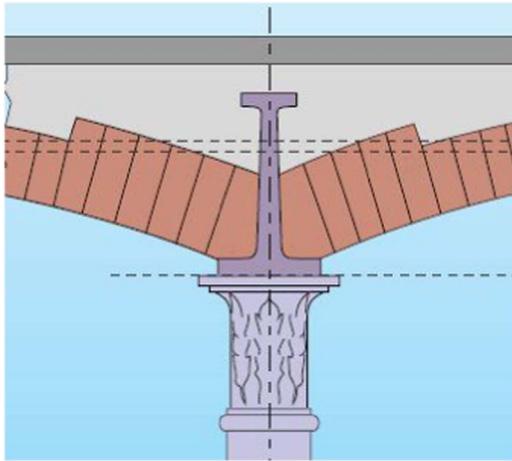


Fig. 1. Typical jack arch beam [2].

The paper presents a reliability analysis in order to estimate appropriate safety factors for fire resistance design of jacked arch cast iron beams. Four different characteristic cross section types have been studied, using randomised stress strain temperature relationships (eight random material property parameters per temperature) in conjunction with a fibre cross section analysis method. From these analyses, the probability distribution of moment capacity has been calculated and material safety factors have been proposed.

The methodology used in this paper is similar with that used successfully by others [10,11] for different types of structure.

## 2. Conditional probability of structural failure in fire

This paper will derive material partial safety factors for cast iron under fire conditions to achieve different levels of reliability, each with a corresponding probability of failure. To estimate the range of acceptable structural failure probabilities in fire, consider building Consequence Class 2 (CC2) according to EN1990 [12] for general building design. This building class is required to achieve a target reliability index of 3.8, corresponding to a probability of failure of  $7.23 \times 10^{-5}$ . This can be taken as the total probability of failure acceptable to the society. When determining the acceptable probability of failure of structures in fire, it is necessary to include the probability of ignition and the probability of flashover given fire occurrence.

### i. Probability of ignition

Several equations have been proposed to quantify the probability of fire occurrence in buildings [13–15]. An example is Poisson distribution of the probability of ignition of  $x$  fires during a time interval  $t$ , as follows [13]:

$$P(X = x) = \frac{1}{x!} \lambda t^x e^{-\lambda t} \quad (1)$$

where  $\lambda$  is the mean fire ignition rate or the average number of fire occurrences per unit time interval and  $X$  is the number of fire occurrences during the time interval  $t$ .

The probability of fire occurrence in building is a function of many parameters (the size of the compartment, the number of compartment etc). Values for  $\lambda$  are given in [16] for several cases. For a 50-year period, considered to be the typical life-time of a building, the probability of fire occurrence in a compartment of 500 m<sup>2</sup> in size ranges from  $10^{-2}$  to 0.2.

### ii. Probability of flashover

Structural resistance is rarely fatally affected before flashover. Therefore, it is usually assumed that structural failure occurs only after flashover. The probability of flashover may be calculated using the following conditional probability equation [10]:

Table 1

Conditional probability of flashover given ignition  $P(\text{flashover} | \text{ignition})$  [17].

Fire protection method	$P(\text{flashover}   \text{ignition})$
Public fire brigade	$10^{-1}$
Sprinkler	$10^{-2}$
High standard fire brigade on site combined with alarm system	$10^{-3}$ – $10^{-2}$
Both sprinkler and high standard residential fire brigade	$10^{-4}$

$$P(\text{fo}) = P(\text{fo} | \text{ignition}) \times P(\text{ignition}) \quad (2)$$

where  $P(\text{fo})$  is the probability of flashover,  $P(\text{fo} | \text{ignition})$  is the conditional probability of flashover given ignition and  $P(\text{ignition})$  is the probability of ignition.

Table 1 gives typical values of conditional probability of flashover given ignition.

Combining with typical values of probability of ignition,  $10^{-2}$  to 0.2 as given in (i), the probability of a flashover fire occurring in a typical building of 50-year life time is between  $2 \cdot 10^{-2}$  and  $10^{-6}$ .

### iii. Probability of structural failure

Combining the above different probability terms, the probability of structural failure in fire is defined as [10]:

$$P(\text{fail}) = P(\text{fail} | \text{fo}) \times P(\text{fo}) \quad (3)$$

where  $P(\text{fail})$  is the probability of structural failure in fire and  $P(\text{fail} | \text{fo})$  is the conditional probability of structural failure in a post-flashover fire.

Therefore, to achieve a target probability of structural failure in fire of  $7.23 \times 10^{-5}$  (corresponding to a reliability index of 3.8), the acceptable conditional probability of structural failure, given a flashover fire, is between  $10^{-3}$  and 1. Clearly a failure probability of 1.0 is not permissible so a minimum safety factor for failure probability of 0.1 is recommended. This paper will estimate the required material partial safety factors for cast iron to achieve conditional probabilities of structural failure of  $10^{-3}$ ,  $10^{-2}$  and  $10^{-1}$  in flashover fires.

## 3. Fire curves

In this paper the standard fire curve [17] has been considered as well as two parametric fire curves [17] for slow ( $O=0.02 \text{ m}^{1/2}$ ,  $b=1120 \text{ J/m}^2 \text{ s}^{1/2} \text{ K}$  and  $q_{t,d}=200 \text{ MJ/m}^2$ ) and fast burning ( $O=0.1 \text{ m}^{1/2}$ ,  $b=1120 \text{ J/m}^2 \text{ s}^{1/2} \text{ K}$  and  $q_{t,d}=200 \text{ MJ/m}^2$ ). The range of difference between the parametric fires from the standard fire is large so that it enables an assessment of whether the findings of the paper would be generally applicable. The considered fire curves, using the parametric fire curve equations in EN 1991-1-2 [17], are shown in Fig. 2.

## 4. Material model

The stress-strain temperature relationships for cast iron are as proposed by the authors in [8] and are illustrated in Fig. 3. The stress-strain diagram parameters in tension are:

- Young's modulus,
- the 0.2% proof stress,
- the maximum stress and the corresponding strain.

For temperatures higher than 400 °C, there is also a descending part in the stress-strain diagram. Therefore, two extra parameters are needed: stress and strain at failure.

Under compression, the stress-strain relationship is simpler than

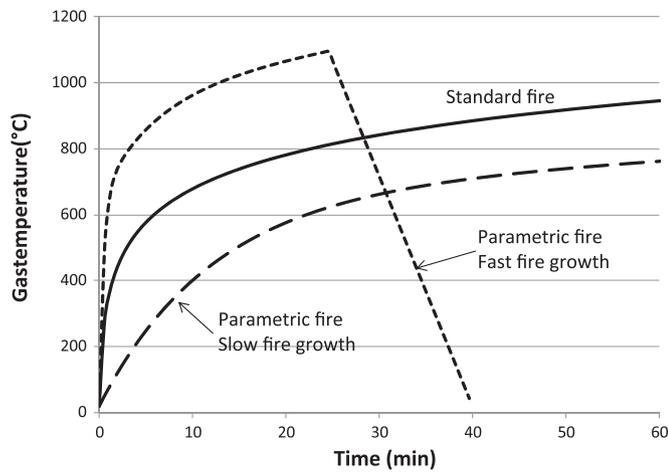


Fig. 2. Parametric (EN 1991-1-2 [17]) and standard fire temperature-time relationships.

that in tension. The required parameters are:

- Young's modulus,
- the proportional limit,
- the 0.2% proof stress and
- the maximum stress and the corresponding strain.

The reduction factors for the Young's modulus, the 0.2% proof stress, the proportional limit and the maximum stress can be modelled according to the reduction factors for steel as defined in EN1993-1-2 [18]. For the remaining parameters, empirical relationships have been proposed by the authors [8].

Assuming normal distribution of variability, based on statistical analysis of available experimental data [8], the mean values and standard deviations of the elevated temperature reduction factors for the various quantities of stress-strain relationship have been estimated. These values are presented in Tables 2–4. Also, typical diagrams of 95% confidence interval vs temperature are presented in Fig. 4. The

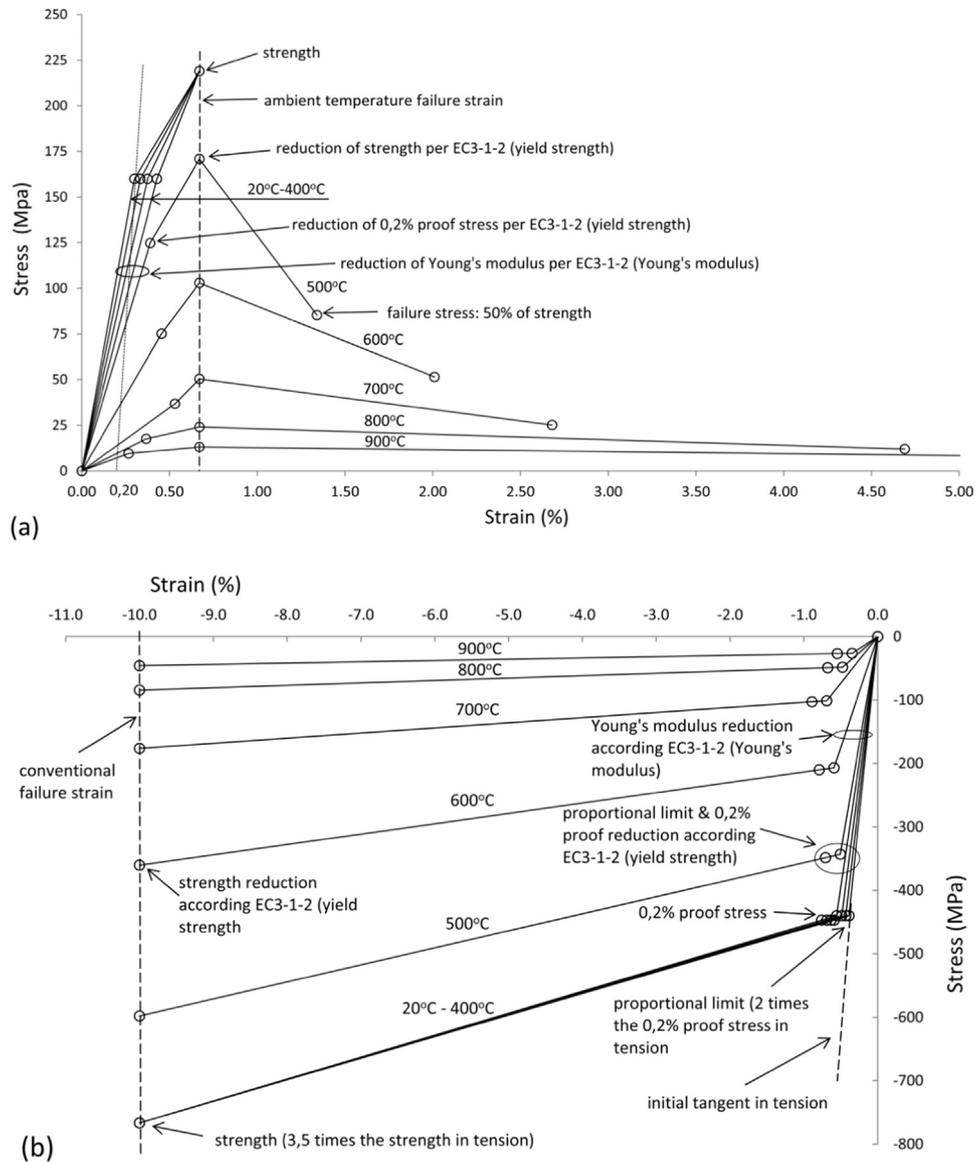


Fig. 3. Stress-strain relationships of cast iron at elevated temperatures, for (a) tension and (b) compression [8].

**Table 2**  
Means and standard deviations for elevated temperature reduction factors in tension.

No	Stress-strain variable	Temperature (°C)	Mean	Standard deviation
1	Young's modulus	100	1.1017	0.1949
2		200	1.0150	0.1088
3		300	1.0250	0.1484
4		400	0.9800	0.1170
5		500	0.8769	0.1239
6		600	0.6070	0.1520
7		700	0.3396	0.1353
8		800	0.1600	0.0762
9		900	0.1138	0.0254
10	0.2% proof stress	100	0.895	0.0495
11		200	0.9250	0.0636
12		300	0.9000	0.0282
13		400	0.9050	0.0353
14		500	0.8247	0.0487
15		600	0.5756	0.1908
16		700	0.3300	0.1199
17		800	0.1455	0.0493
18		900	0.0793	0.0178
19	Maximum stress	100	0.9658	0.1523
20		200	0.9873	0.1388
21		300	0.9896	0.1033
22		400	0.9693	0.1307
23		500	0.8687	0.0640
24		600	0.6351	0.1685
25		700	0.4891	0.1424
26		800	0.3026	0.1668
27		900	0.1195	0.0678
28	Strain at maximum stress	100	0.97611	0.339701
29		200	1.00597	0.356418
30		300	1.12835	0.310448
31		400	1.23880	0.437463
32		500	1.04850	0.620896
33		600	1.02238	0.536567
34		700	1.41791	0.581194
35		800	0.62194	0.430896
36		900	0.56223	0.161045

**Table 3**  
Means and standard deviations for failure strain in tension.

No	Variable (strain (%))	Temperature (°C)	Mean	Standard deviation
1	Failure strain	500	0.8275	0.5639
2		600	1.7429	0.7251
3		700	2.861	0.8466
4		800	3.7126	0.7079
5		900	5.6080	1.9608

uncertainties are very large because of the nature of the material. Cast iron is a brittle material and its mechanical properties are affected by the method of casting, including cooling and solidification. The most important parameter, which severely affects its mechanical properties (especially in tension), is the existence of randomly distributed graphite flakes within its mass. These graphite flakes are microstructure flaws. Depending on the position, the number and the size of these flows (discontinuities), cast iron may have very different properties in tension. No one specimen is the same as the other, as they have different flaws. It is therefore important that the necessary material safety factors are quantified to ensure that cast iron structures achieve acceptable levels of reliability in fire.

Due to a lack of data, the mechanical properties presented in Tables 2–4 are considered to be independent variables.

**Table 4**  
Means and standard deviations for elevated temperature reduction factors in compression.

No	Stress-strain variable	Temperature (°C)	Mean	Standard deviation
1	Young's modulus	100	0.9999	0.0698
2		200	1.0084	0.0249
3		300	0.9542	0.1015
4		400	0.8868	0.0739
5		500	0.6933	0.0306
6		600	0.4967	0.0208
7		700	0.2933	0.0351
8		800	0.0983	0.0125
9		900	0.0740	0.0085
10	Proportional limit	100	1.0003	0.0186
11		200	0.9934	0.0055
12		300	0.9855	0.0111
13		400	0.9652	0.0220
14		500	0.8220	0.0089
15		600	0.4033	0.0152
16		700	0.1461	0.0016
17		800	0.0589	0.0049
18		900	0.0337	0.0058
19	0.2% proof stress	100	0.9662	0.0449
20		200	0.9637	0.0398
21		300	0.9718	0.0344
22		400	0.9339	0.0483
23		500	0.6789	0.0222
24		600	0.3121	0.0398
25		700	0.1752	0.0037
26		800	0.0969	0.0081
27		900	0.0553	0.0092

**5. Calculation of bending moment capacity: fibre analysis model**

The Monte-Carlo method has been used to evaluate the material partial safety factors for cast iron beams at elevated temperatures. To facilitate this calculation, a quick and simplified method should be developed to calculate the bending moment capacity of cast-iron beam cross-section. A fibre analysis model, based on [19,20], has been developed and validated against detailed finite element analysis [9]. A schematic presentation of the fibre model is shown in Fig. 5. A summary of the method is presented below:

At a curvature  $k$ :

1. The initial position of the neutral axis is assumed to be at the centre of gravity.
2. The cross-section is divided into a large number of fine layers.
3. The strain at the mid-depth of each layer is calculated.
4. The temperature at the mid-depth of each layer is calculated.
5. The stress at the mid-depth of each layer is calculated.
6. The force of each layer is calculated.
7. The tensile ( $F_t$ ) and the compressive forces ( $F_c$ ) of all layers are summed.
8. If  $|F_t - F_c| / F_t < r$ , where  $r$  is a small value (taken as 0.001 in this research), the corresponding moment ( $M$ ) is calculated.
9. If  $|F_t - F_c| / F_t > r$ , the algorithm returns to step 1 and the position of the neutral axis is modified according to the equation  $y_{n+1} = y_n - ((F_t - F_c) / (F_t + F_c)) * y_{CG}$  (where  $y$  is the distance from the bottom of the cross section and  $y_{CG}$  is the distance of the centre of gravity from the bottom of the cross section).
10. If increasing the curvature gives a smaller bending moment, then the ( $M, k$ ) result of the previous iteration is the first point of the descending branch of the moment-curvature curve, and the corresponding bending moment is the final (maximum) bending moment capacity of the beam.

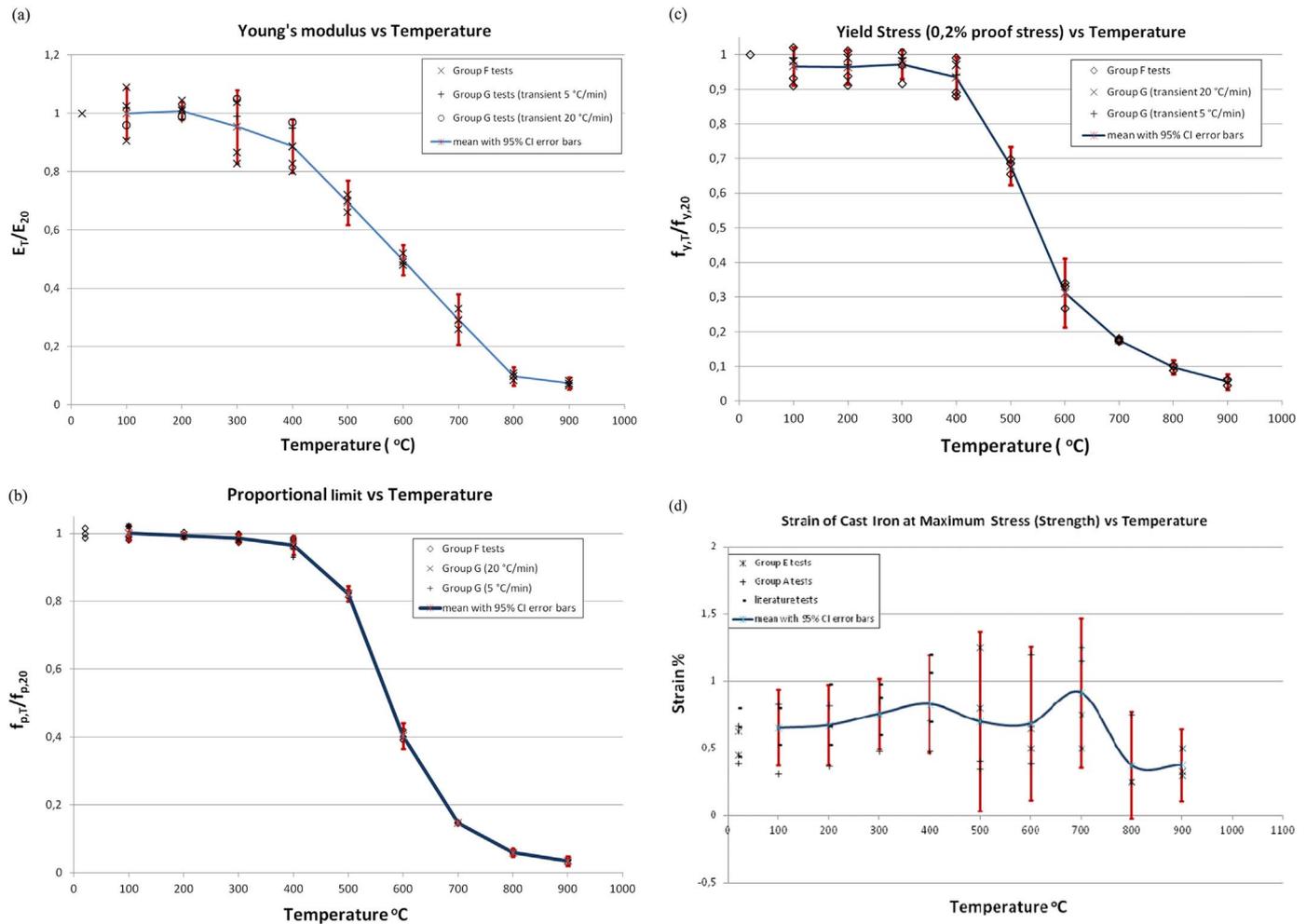


Fig. 4. Scatters and 95% confidence interval (CI) vs temperature for key mechanical properties of cast iron in tension and compression (a) Young's modulus in compression, (b) proportional limit in compression, (c) 0.2% proof stress in compression, (d) strain at tensile strength, (e) failure strain in tension, (f) 0.2% proof stress in tension, (g) strength in tension and (h) Young's modulus in tension. Based on the test data of Ref. [8].

### 6. Cross sections

Four cast iron cross sections were used for the analysis and they are shown in Fig. 6. The first cross section (Fig. 6a), used in the Marshall Mill [21], is short and thin. Its section factor is low (perimeter length/cross-section area for the bottom flange), so when it is exposed to fire, it would increase temperatures rapidly. Also because it is shallow, the cross-section temperature distribution would be relatively uniform. The last cross section (Fig. 6d) is tall and thick. Therefore, it has a low section factor and is expected to increase its temperature slowly. Also it would experience large temperature differences in the cross-section. The cross sections in Fig. 6b and c are intermediate cross sections between the previous two. The nominal dimensions of the cross sections are used in the analysis without taking into account their variations because the focus of this study is on material partial safety factors.

### 7. Temperature profiles

The sections were assumed to be exposed to the standard or natural (parametric) fire [17] as presented in Section 3 and the thermal profiles of the cross-sections were calculated using the finite element software ABAQUS. Fig. 7 shows the thermal boundary conditions and material properties used. The thermal properties of cast iron are those of steel according to EN1993-1-2 [18] and the thermal properties of the insulation are those of concrete according to EN1992-1-2 [22] as

proposed by the authors in [5–7]. The moment capacity of cast iron is not sensitive to the variations of thermal properties of both the insulation materials and cast iron as found in the previous sensitivity study of the authors [7]. Therefore, they are not considered as random variables in this study.

The temperature profiles of the sections were used as input in subsequent calculations of bending moment resistances of the cross-sections. This paper will present results for 30 and 60 min of the standard fire exposure and for parametric fires. Under parametric fires, the minimum bending moment capacities were calculated.

Cast iron beams are simply supported at ends, therefore, it is reasonable to assume that temperature distribution along the length of cast iron beams is uniform, in accordance with accepted design practice.

### 8. Methodology of reliability analysis

Monte-Carlo simulations were performed to estimate the probability of failure of the cast-iron beam cross-sections and the corresponding material safety factors.

The material safety factor is calculated by the following equation:

$$\gamma_{M,fi} = \frac{M_{fi,T}}{M_{fi,T}^{Pf}} \quad (4)$$

where  $M_{fi,T}$  is the moment capacity calculated using the nominal cast-iron mechanical property model in [8].  $M_{fi,T}^{Pf}$  is the moment capacity

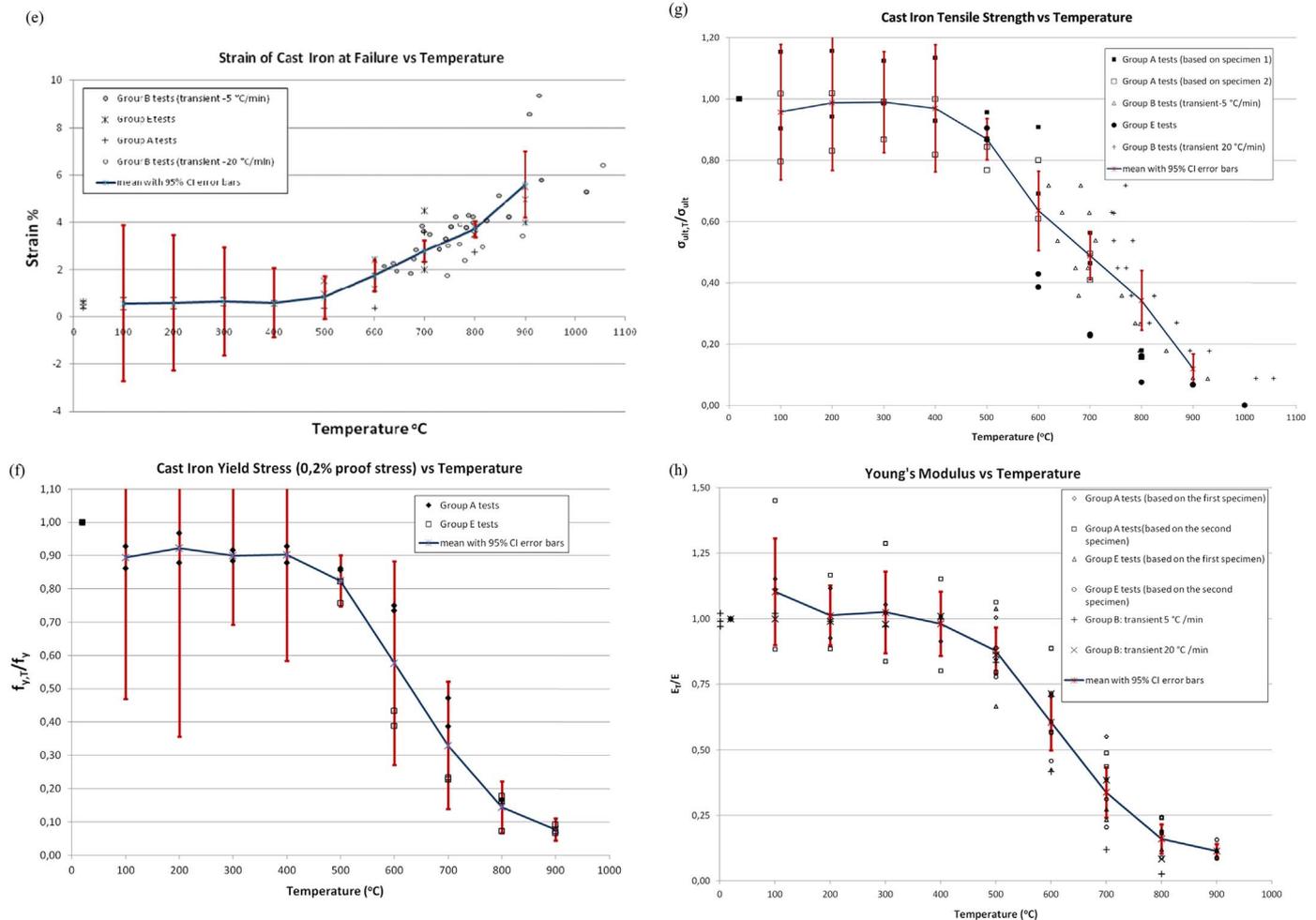


Fig. 4. (continued)

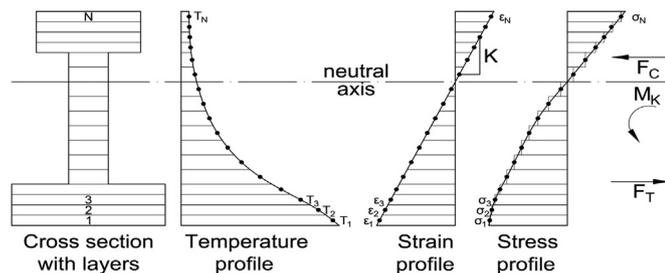


Fig. 5. Schematic presentation of the fibre analysis procedure to obtain cast iron beam bending moment capacity [9].

corresponding to the target conditional probability of failure  $P_f$  ( $10^{-3}$ ,  $10^{-2}$ ,  $10^{-1}$ ) at the standard fire exposure time  $T$  or the minimum moment capacity during a parametric fire, obtained to satisfy the following probability condition:

$$M_{d,\tilde{n}}^{P_f} = M_{d,\tilde{n}} | P_f \quad (5)$$

where  $M_{d,\tilde{n}}$  is moment capacity distribution calculated using randomised stress-strain curves of cast iron.

In the Monte Carlo simulations, the following nine elevated temperature mechanical properties of cast iron were varied:

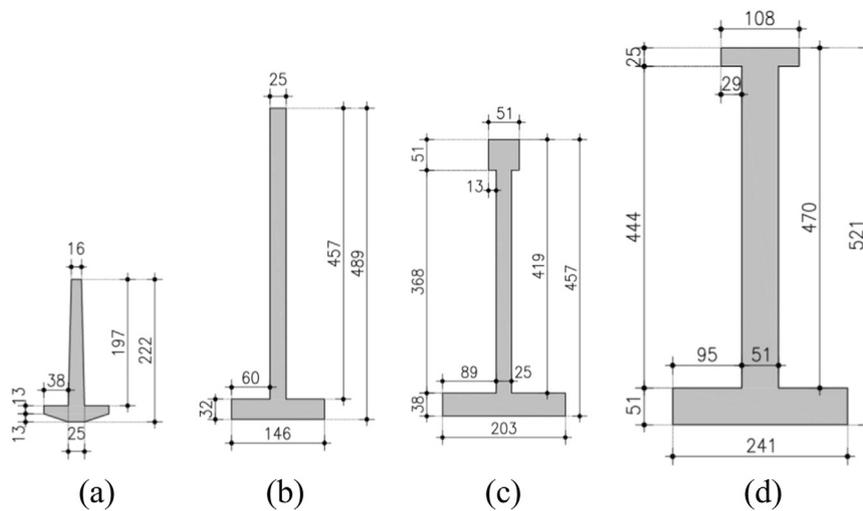
- Young's modulus in tension
- 0.2% proof stress in tension
- maximum tensile stress
- strain corresponding to the maximum tensile stress

- strain at failure in tension
- Young's modulus in compression
- proportional limit in compression
- 0.2% proof stress in compression
- maximum compressive stress

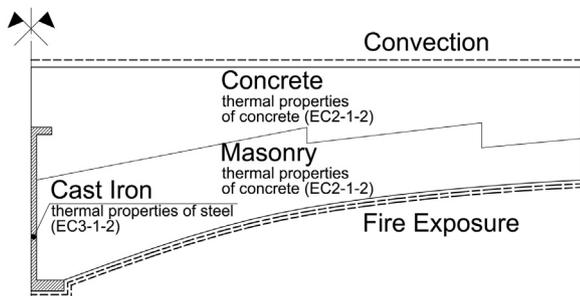
The mean and standard deviation values for these variables are given in Tables 2–4.

The Monte Carlo simulation procedure is outlined below:

- For each Monte Carlo simulation, random values of the above nine variables at the corresponding temperatures were generated according to their distributions, assumed to be normal with the mean and standard deviation values in Tables 2–4. A total of 100,000 simulations were run, based on the rule of thumb [23] that the sample size should exceed  $10/P_f$ , where the smallest  $P_f$  considered ( $10^{-3}$ ). Section 9 further confirms that this total number is adequate.
- Any negative property value was rejected.
- After selecting the nine random mechanical properties of cast iron, the stress-strain temperature relationships were generated.
- Use the elevated temperature stress-strain temperature relationships, for a given cross section and temperature profile, the moment resistance was calculated using the fibre analysis model outlined in Section 5.
- From the calculated moment capacity results, the normal distribution parameters (mean, standard deviation) were calculated. Fig. 8 shows typical results for Shaw's H cross section for 30 min of the standard fire exposure.



**Fig. 6.** Cast iron cross section types used in the analysis, based on [21] (a) Marshall mill (1817), jack arch span 3.35 m, (b) Armley mill (1823), jack arch span 2.60 m, (c) Shaw's G mill (1851), jack arch span 2.44 m and (d) Shaw's H mill (1880), jack arch span 2.75 m.



**Fig. 7.** Thermal boundary conditions and thermal properties of materials used for the thermal analysis.

- From the calculated moment capacity distribution, the corresponding moment capacities for  $P_f=10^{-1}$ ,  $10^{-2}$  and  $10^{-3}$  are calculated (Eq. (5), based on the cumulative probability density of bending moment capacity). Fig. 9 shows an example of this procedure.

**9. Convergence of reliability analysis – sample size**

To confirm the rule of thumb [23] for the necessary number of simulations, the cross sections in Fig. 8a and d were used. The thermal profile was that at 30 min of standard fire exposure [16]. The probability of failure was  $10^{-3}$ . The error E is defined by the following equation:

$$E = \left| \frac{M_{f,T}^{i=100,000} - M_{f,T}^i}{M_{f,T}^{i=100,000}} \right| \tag{6}$$

where i is the sample size and i=100,000 is the sample size used for the analysis within the paper. Fig. 10 represents the error vs sample size for the two cross sections for the given probabilities of failure. In all cases, a sample size much less than 100,000 is sufficient.

**10. Results**

Tables 5–8 present results of the reliability analysis.

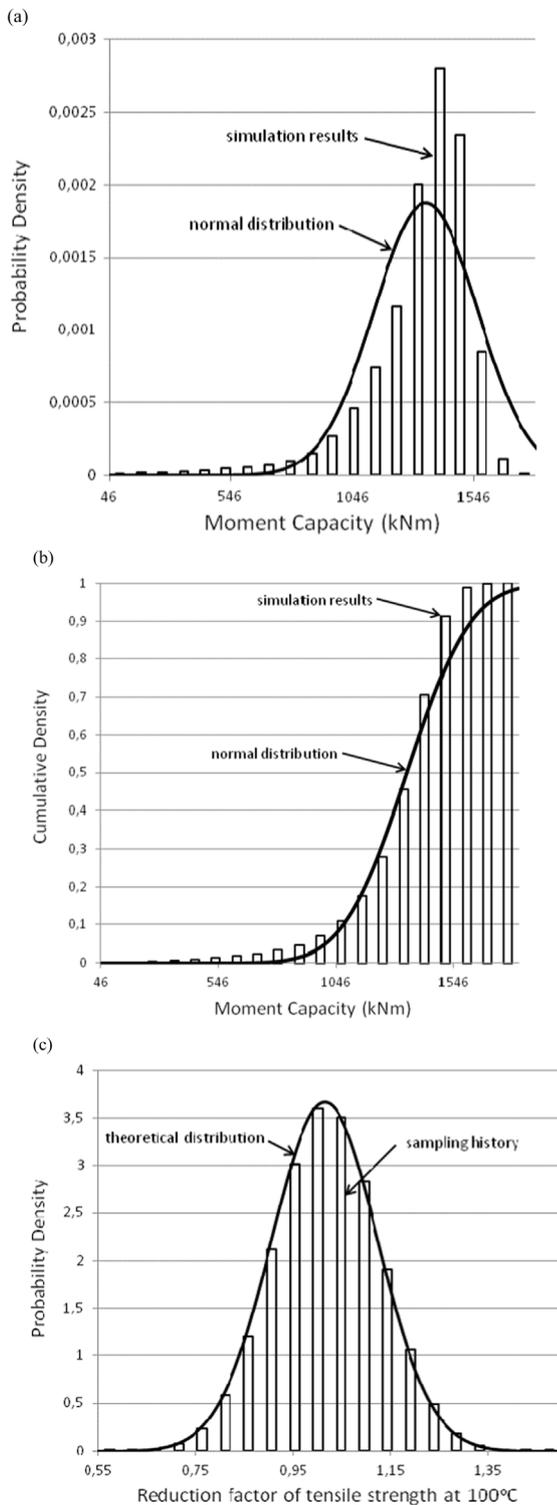
From these results the short cross section (Fig. 6a) needs higher material safety factors than the tall cross section (Fig. 6d). This is expected as the short cross-section has relatively uniform temperature distribution, therefore uncertainties in cast-iron mechanical properties

affect a large part of the short cross section. In contrast, just a short part of the tall cross section experiences elevated temperatures.

The proposed material safety factors are high compared to the proposed values in Eurocodes for modern steel. This is expected, because the production and quality control of modern steel follow much more strict specifications than the cast iron beams manufactured during the 19th century when the production technology and quality control were more primitive.

For the standard fire exposure, the safety factors for the higher fire rating, R60 are slightly higher than for the lower fire rating, R30. This is due to the larger scatter of tensile properties at higher temperatures associated with the higher fire rating. However, the differences in the material safety factors for the two different fire ratings with the same probability of failure are relatively small. It is therefore possible to use the same material safety factor for different fire ratings. The safety factor to reach a failure probability of  $10^{-3}$ , being the likely lowest target probability to achieve a reliability index of 3.7, ranges from 4.19 to 5.53. This is very close to the ambient temperature safety factor of 5.0 [24]. The safety factors for the deeper Shaw's sections tend to be lower than those for the shallower Marshall cross-section. Again, this may be explained by the more uniform and higher temperatures, which are attained in the shallower Marshall cross-section. However, again the differences in the safety factors for the two beam sections are relatively small. To summarise, it is possible to recommend one set of material safety factors according to the target probability of failure, for different fire ratings and cross-section types. Approximately, the following safety factors may be used: 1.5, 2.5 and 4.5 for target probabilities of  $10^{-1}$ ,  $10^{-2}$  and  $10^{-3}$  respectively.

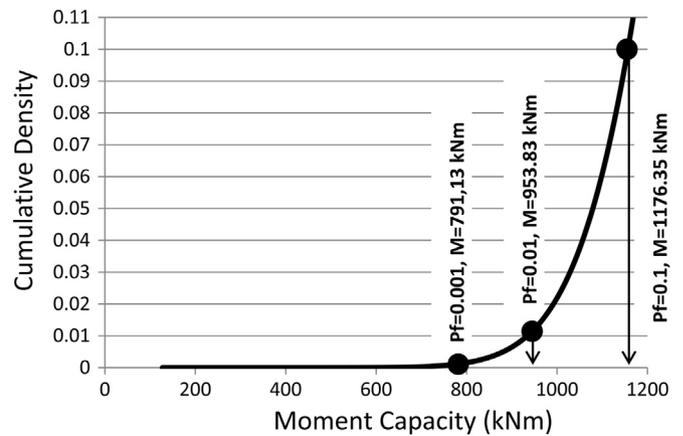
Under the parametric fire conditions, the safety factors for both the slow and fast fires are similar, with the safety factors for the slow fire being slightly higher than those for the fast fire. The safety factors for 60 min of standard fire resistance are between those for the slow and fast fires. This may be explained by the larger variations in cast iron mechanical properties at higher temperatures (given similar lower flange temperatures) in cast iron cross-sections under different fire conditions. Fig. 11 shows the temperature distributions in a typical cast iron cross-section for different fires at the fire exposure times when the design bending moments (either at the end of standard fire resistance time, or when the minimum bending resistance is reached under the parametric fires) are calculated. The different safety factors (slow fire > 60 min standard fire > fast fire) are in accordance with the cross-section temperatures under different fires (slow fire > 60 min of



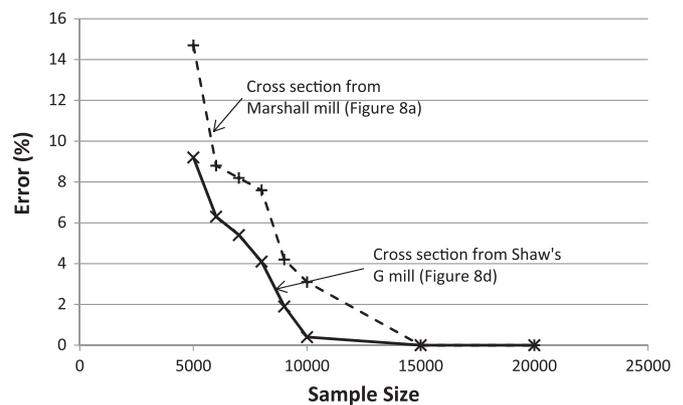
**Fig. 8.** Typical probability distributions for mechanical property and bending moment capacity (for Shaw's H cross section exposed for 30 min standard fire exposure) (a) probability density of moment capacity (b) cumulative probability of moment capacity and (c) sampling history vs theoretical distribution of the reduction factor of tensile strength at 100 °C.

standard fire > fast fire, given the same lower flange temperature). Nevertheless, the safety factors for the different fire scenarios are similar and one set of safety factors may be used for different fires.

Considering that the safety values are high, to enable one sensibly



**Fig. 9.** Moment capacities for corresponding cumulative failure probabilities ( $10^{-1}$ ,  $10^{-2}$ ,  $10^{-3}$ ) for Shaw's H cross section exposed for 30 min standard fire exposure.



**Fig. 10.** Error vs sample size for  $P_f=10^{-3}$ .

rounded single value of safety factor to be used for different fire conditions, safety factor values of 1.5, 3.0 and 5.0 are recommended for target probabilities of failure of  $10^{-1}$ ,  $10^{-2}$ ,  $10^{-3}$  respectively.

### 11. Conclusions

This paper has presented the results of a Monte-Carlo simulation to derive material safety factors for cast iron beams. In this study, the beam bending moment capacity was calculated using a fibre analysis method. The mean and standard deviation values for the different key properties of the stress-strain-temperature relationships of cast iron in both tension and compression (Young's modulus in tension, the 0.2% proof stress in tension, the maximum tensile stress, the strain corresponding to the maximum tensile stress, the strain at failure in tension, Young's modulus in compression, the proportional limit in compression, the 0.2% proof stress in compression, the maximum compressive stress), were estimated from an assessment of a large amount of data collected by the authors, including the authors' own elevated temperature test data. These properties were treated as independent variables due to a complete lack of data on their correlation.

Based on an analysis of the probability of fire occurrence and the conditional probability of flashover given fire occurrence, the target conditional beam failure probability given flashover was found to be in the range of  $1.0 \cdot 10^{-3}$  to achieve a reliability index of 3.8.

The safety factors tend to be higher for high standard fire rating, slow parametric fire and shallow cast iron section. However, these variables have relatively minor influences on the safety factors.

**Table 5**  
Material safety factors for Marshall's cross section (Fig. 6a).

Probability of failure $P_f$	Standard fire [17]				Natural – parametric fires [17]			
	Moment capacity (kN m) after standard fire exposure time of		Safety factor $\gamma_{M,fi}$ for		Minimum moment capacity (kN m) and safety factor for slow fire	Safety factor $\gamma_{M,fi}$ for slow fire	Minimum moment capacity (kN m) and safety factor for fast fire	Safety factor $\gamma_{M,fi}$ for fast fire
	$M_{fi,T}^{P_f}$							
	30 min	60 min	30 min	60 min				
$10^{-1}$	58.64	28.02	1.52	1.58	21.22	1.59	28.97	1.55
$10^{-2}$	35.13	15.33	2.54	2.89	11.63	2.90	16.15	2.78
$10^{-3}$	24.66	8.01	3.62	5.53	6.08	5.55	8.26	5.44
$M_{fi,LT}$	89.34	44.37			33.74		44.91	
Material model [8]								

**Table 6**  
Material safety factors for Armley's cross section (Fig. 6b).

Probability of failure $P_f$	Standard fire [17]				Natural – parametric fires [17]			
	Moment capacity (kN m) after standard fire exposure time of		Safety factor $\gamma_{M,fi}$ for		Moment capacity (kN m) after natural fire exposure time for minimum moment capacity of	Safety factor $\gamma_{M,fi}$ for slow fire	Minimum moment capacity (kN m) and safety factor for fast fire	Safety factor $\gamma_{M,fi}$ for fast fire
	$M_{fi,T}^{P_f}$							
	30 min	60 min	30 min	60 min				
$10^{-1}$	421.40	251.09	1.40	1.51	233.87	1.51	260.35	1.50
$10^{-2}$	304.10	140.42	1.94	2.70	128.42	2.75	144.64	2.70
$10^{-3}$	209.95	76.91	2.81	4.93	71.06	4.97	79.54	4.91
$M_{fi,LT}$	589.96	379.15			353.15		390.52	
Material model [8]								

**Table 7**  
Material safety factors for Shaw's G cross section (Fig. 6c).

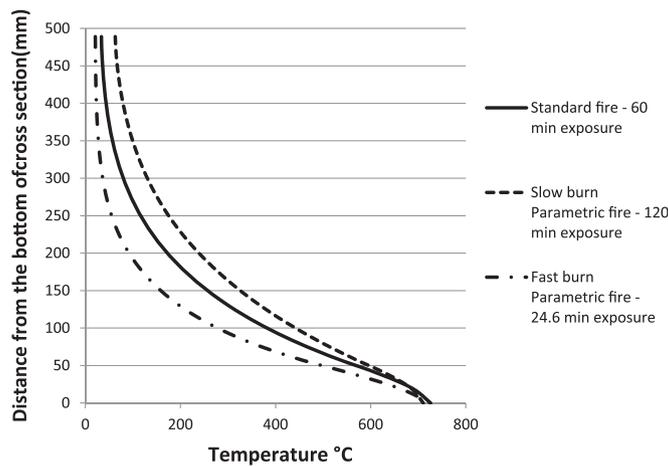
Probability of failure $P_f$	Standard fire [17]				Natural – parametric fires [17]			
	Moment capacity (kN m) after standard fire exposure time of		Safety factor $\gamma_{M,fi}$ for		Moment capacity (kN m) after natural fire exposure time for minimum moment capacity of	Safety factor $\gamma_{M,fi}$ for	Minimum moment capacity (kN m) and safety factor for fast fire	Safety factor $\gamma_{M,fi}$ for fast fire
	$M_{fi,T}^{P_f}$							
	30 min	60 min	30 min	60 min				
$10^{-1}$	566.69	295.58	1.36	1.49	272.80	1.50	307.05	1.47
$10^{-2}$	428.17	169.38	1.80	2.60	155.00	2.64	174.95	2.58
$10^{-3}$	282.30	90.62	2.73	4.86	83.34	4.91	92.87	4.86
$M_{fi,LT}$	770.70	440.41			409.20		451.37	
Material model [8]								

Therefore, it is recommended that to achieve the target conditional probability of  $10^{-1}$ ,  $10^{-2}$ ,  $10^{-3}$ , the approximate safety factors are 1.50, 3.0 and 5.0 respectively. The same material safety factors may be used for different cast iron beam cross-sections, different standard fire

ratings and parametric fires. The fire protection engineer should determine the target conditional probability, based on analysis of the probability of fire occurrence and the conditional probability of flash-over given fire occurrence.

**Table 8**  
Material safety factors for Shaw's H cross section (Fig. 6d).

Probability of failure $P_f$	Standard fire [17]				Natural – parametric fires [17]				
	Moment capacity (kN m) after standard fire exposure time of		Safety factor $\gamma_{M,fi}$ for		Moment capacity (kN m) after natural fire exposure time for minimum moment capacity of		Safety factor $\gamma_{M,fi}$ for		Minimum moment capacity (kN m) and safety factor for fast fire
	$M_{f,T}^{Pf}$				$M_{f,T}^{Pf}$				
	30 min	60 min	30 min	60 min					
$10^{-1}$	1176.35	779.40	1.33	1.44	701.56		1.44	881.69	1.40
$10^{-2}$	953.83	483.68	1.64	2.32	433.58		2.33	539.02	2.29
$10^{-3}$	791.13	267.47	1.98	4.19	241.11		4.19	586.83	4.17
$M_{f,LT}$	1565.73	1122.39			1010.25			1234.36	
Material model [8]									



**Fig. 11.** Temperature profile developed within the Armley cross section (Fig. 6b) for 60 min of standard fire exposure compared with the temperature profiles (at the minimum moment capacity) for natural fires.

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