

# **PROBABILISTIC DETERMINATION OF MATERIAL SAFETY FACTOR FOR CAST IRON BEAMS IN JACK ARCHED CONSTRUCTION EXPOSED TO FIRE**

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## **ABSTRACT**

Cast iron beams were used in many 19th century structures, especially in fireproof flooring systems (such as jack arch). Many such structures are still in use today and it is important that they fulfil the current requirements on fire resistance when there is a change of use. These structures are out of scope of the modern design codes and the old design codes do not provide guidance for fire resistance design. Furthermore, cast iron is a brittle material weak in tension, and there are many uncertainties in its mechanical properties at ambient and elevated temperatures due to material flaws. Based on extensive literature survey and fresh test data, the authors have proposed elevated temperature stress-strain-temperature relationships for cast iron, based on the EC3-1-2 reduction factors for carbon steel. Although the proposed relationships are reasonably close to the gathered data, there are considerable scatters and it is necessary to quantify the probability of structural failure when using such relationships, and to introduce safety factors to reduce the probability of structural failure in fire to an acceptable level. This paper presents the results of a study whose purpose is to derive an appropriate safety factor for fire safety design of cast iron beams. In this study, a fibre analysis method has been developed to calculate the moment capacities of two different types of cast iron cross sections. Using randomized stress-strain-temperature relationships, based on the variability of the different governing parameters (maximum stress, 0.2% proof stress, corresponding strains at maximum stress (strength) and failure and Young's modulus under tension, Young's modulus, proportional limit, 0.2% proof stress and the maximum stress under compression), the probability distribution of moment capacity has been calculated. Based on the criterion of cast iron beam failure not exceeding probabilities of  $10^{-1}$ ,  $10^{-2}$ ,  $10^{-3}$  and  $10^{-4}$ , material safety factors of 1.5, 2.5, 4.5 and 5.5 respectively have been obtained.

## **1. INTRODUCTION**

Many 19th century historic buildings throughout the UK, Central and Western Europe as well as the US were built with cast iron structural elements, as main loadbearing columns and beams, especially during the period of 1820-1850 [1]. Cast iron beams are typically partially fire protected using various types of thermal insulation systems ([2], [3], [4], with the jack arch floor, as illustrated in Figure 1, being the most widely applied. Because of limited use of cast iron structures in modern construction, there has been very limited research on cast iron structures, at ambient temperature and in fire.

Cast iron structural beams exhibit different behavior from that of modern steel beams. When cast iron beams are used as part of the jack arch construction, the temperature distribution in the cast iron cross-section is severely non-uniform. Also, the stress-strain curve of cast iron does not possess the same degree of plastic behavior of steel, which makes analyzing cast iron beams using the plastic analysis method not possible.

Furthermore, cast iron behaves differently under tension and compression.

Based on extensive assessments of thermal and mechanical properties of cast iron and associated insulation materials at ambient and elevated temperatures [5], [6], [7], and new experimental data by the authors [8], the authors have proposed thermal properties for the relevant thermal insulation materials, and thermal and mechanical properties for cast iron [5], [6], [7], including the thermal expansion coefficient and stress-strain-temperature relationships [8]. More recently, the authors have developed a simplified method to calculate the moment capacity of jack arch beam cross-sections at elevated temperatures [9]. Because of the uncertainties in the various material properties, there is a need to develop material safety factors for fire safety design of cast-iron structures. This is the aim of this paper.

The paper presents a reliability analysis in order to estimate appropriate safety factors for fire design of arch jacked cast iron beams. Two different characteristic cross sections have been studied and a randomised stress strain temperature relationship

(eight random parameters per temperature) in conjunction with a fibre cross section analysis method has been used. From these analyses the probability distribution of moment capacity has been calculated and material safety factors have been proposed.

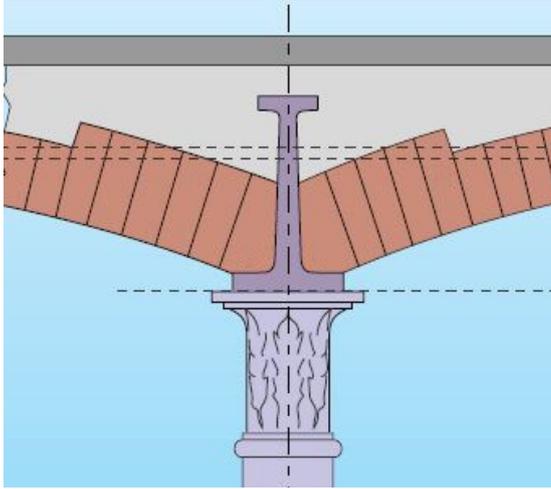


Figure 1. Typical jack arch beam [10].

## 2. METHODOLOGY

The required safety level for general building design, according to EN1990 [11], is to achieve a target reliability index of 3.8, corresponding to a probability of failure of  $7.23 \times 10^{-5}$ . This is the total probability of failure. When applied to fire safety design, it is necessary to include the probability of ignition and the probability of flashover given fire occurrence.

### i. Probability of fire ignition

Several equations have been proposed to quantify the probability of fire occurrence in buildings [12], [13], [14]. An example is Poisson distribution of the probability of ignition of  $x$  fires in a time interval  $t$ , as follows [12]:

$$P(X = x) = \frac{1}{x!} \lambda t^x e^{-\lambda t} \quad (1)$$

where  $\lambda$  is the mean fire ignition rate or the average number of fire occurrence per unit time interval and  $X$  is the number of fire occurrences during the time interval  $t$ .

The probability of fire occurrence in a building is a function of many parameters (the size of the compartment, the number of compartment etc).

Values for  $\lambda$  are given in [15] for several cases. For a 50-year period, considered to be the typical life-time of a building, the probability of fire occurrence in a compartment of 500 m<sup>2</sup> in size ranges from  $10^{-2}$  to 0.2.

### ii. Probability of flashover

Structural resistance is rarely fatally affected before flashover. Therefore, it is usually assumed that structural failure occurs only after flashover. The probability of flashover may be calculated using the following conditional probability equation [16]:

$$P(fo) = P(fo|ignition) \times P(ignition) \quad (2)$$

where  $P(fo)$  is the probability of flashover,  $P(fo|ignition)$  is the conditional probability of flashover given ignition and  $P(ignition)$  is the probability of ignition.

Table 1 gives typical values of the probability of flashover given ignition.

Combining with typical values of probability of ignition,  $10^{-2}$  to 0.2 as given in (i), the probability of a flashover fire in a typical building of 50-year life time is between  $2 \cdot 10^{-2}$  and  $10^{-6}$ .

### iii. Probability of structural failure

Combining the above different probability terms, the probability of structural failure in fire is defined as [16]:

$$P(fail) = P(fail|fo) \times P(fo) \quad (3)$$

where  $P(fail)$  is the probability of structural failure in fire and  $P(fail|fo)$  is the probability of structural failure in a post-flashover fire.

Therefore, to achieve a target probability of structural failure in fire of  $7.23 \times 10^{-5}$  (corresponding to a reliability index of 3.8), the acceptable probability of structural failure, given a flashover fire, is between  $10^{-3}$  and 1. This paper will estimate the required material safety factors to achieve these probabilities of structural failure in flashover fires.

**Table 1. Probability of flashover given ignition P(flashover | ignition) [15]**

Protection method	P(flashover   ignition)
Public fire brigade	$10^{-1}$
Sprinkler	$10^{-2}$
High standard fire brigade on site combined with alarm system	$10^{-3}$ to $10^{-2}$
Both sprinkler and high standard residential fire brigade	$10^{-4}$

### 3. MATERIAL MODEL

The stress-strain temperature relationships for cast iron are as proposed by the authors in [8] and are illustrated in Figure 2. The stress-strain diagram parameters in tension are:

- Young's modulus,
- the 0.2% proof stress,
- the maximum stress and the corresponding strain.

For temperatures higher than 400°C, there is also a descending part in the stress-strain diagram. Therefore, two extra parameters are needed: stress and strain at failure.

Under compression, the stress-strain relationship is simpler than that in tension. The required parameters are:

- Young's modulus,
- the proportional limit,
- the 0.2% proof stress and
- the maximum stress and the corresponding strain.

The reduction factors for the Young's modulus, the 0.2% proof stress, the proportional limit and the maximum stress can be modelled according to the reduction factors for steel as defined in EN1993-1-2 [17]. For the remaining parameters, empirical relationships have been proposed by the authors [8].

Assuming normal distribution of variability, based on statistical analysis of available experimental data [8], the mean values and standard deviations of the elevated temperature reduction factors for the various values of the stress-strain relationship have been estimated. These values are presented in Tables 2 to 4. Also, typical diagrams of 95%

confidence interval vs temperature are presented in Figure 4.

### 4. CALCULATION OF BENDING MOMENT CAPACITY: FIBRE ANALYSIS MODEL

The Monte-Carlo method will be used to evaluate the material safety factors for cast iron beams at elevated temperatures. To facilitate this calculation, a quick and simplified method should be developed to calculate the bending moment capacity of cast-iron beam cross-section. A fibre analysis model, based on [19] and [20], has been developed and validated against detailed finite element analysis [9]. A schematic presentation of the fibre model is shown in Figure 3. A summary of the method is presented below:

At a curvature  $k$ :

1. The initial position of the neutral axis is assumed to be at the centre of gravity.
2. The cross-section is divided into a large number of fine layers.
3. The strain at the mid-depth of each layer is calculated.
4. The temperature at the mid-depth of each layer is calculated.
5. The stress at the mid-depth of each layer is calculated.
6. The force of each layer is calculated.
7. The tensile ( $F_t$ ) and the compressive forces ( $F_c$ ) of all layers are summed.
8. If  $|F_t - F_c| / F_t < r$ , where  $r$  is a small value (taken as 0.001 in this research), the corresponding moment ( $M$ ) is calculated.
9. If  $|F_t - F_c| / F_t > r$ , the algorithm returns to step 1 and the position of the neutral axis is modified according to the equation  $y_{n+1} = y_n - ((F_t - F_c) / (F_t + F_c)) * y_{CG}$  (where  $y$  is the distance from the bottom of the cross section and  $y_{CG}$  is the distance of the centre of gravity from the bottom of the cross section).
10. If increasing the curvature gives a smaller bending moment, then the ( $M$ ,  $k$ ) result of the previous iteration is the first point of the descending branch of the moment-curvature curve, and the bending moment is the final bending moment capacity of the beam.

## 5. CROSS SECTIONS

Two cast iron cross sections were used for the analysis and they are shown in Figure 4. The first cross section (Figure 4a), used in the Marshall Mill [21], is short and thin. Its section factor is low (perimeter length/cross-section area for the bottom flange), so when it is exposed to fire, it would increase temperatures rapidly. Also because it is

shallow, the cross-section temperature distribution would be relatively uniform. The second cross section (Figure 4b) is tall and thick. Therefore, it has a high section factor and is expected to increase its temperature slowly. Also it would experience large temperature differences in the cross-section.

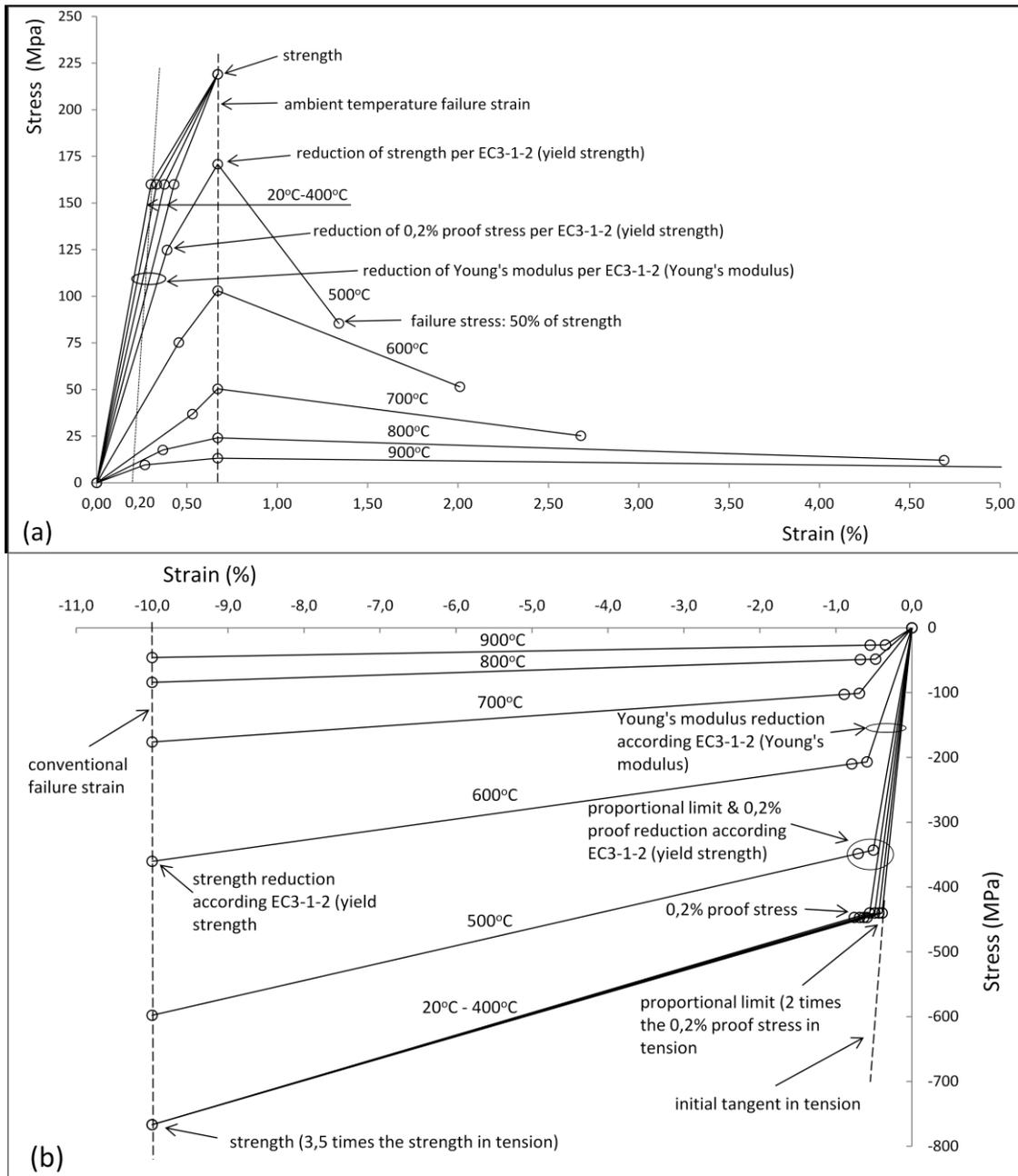


Figure 2. Stress-strain relationships of cast iron at elevated temperatures, for (a) tension and (b) compression [8].

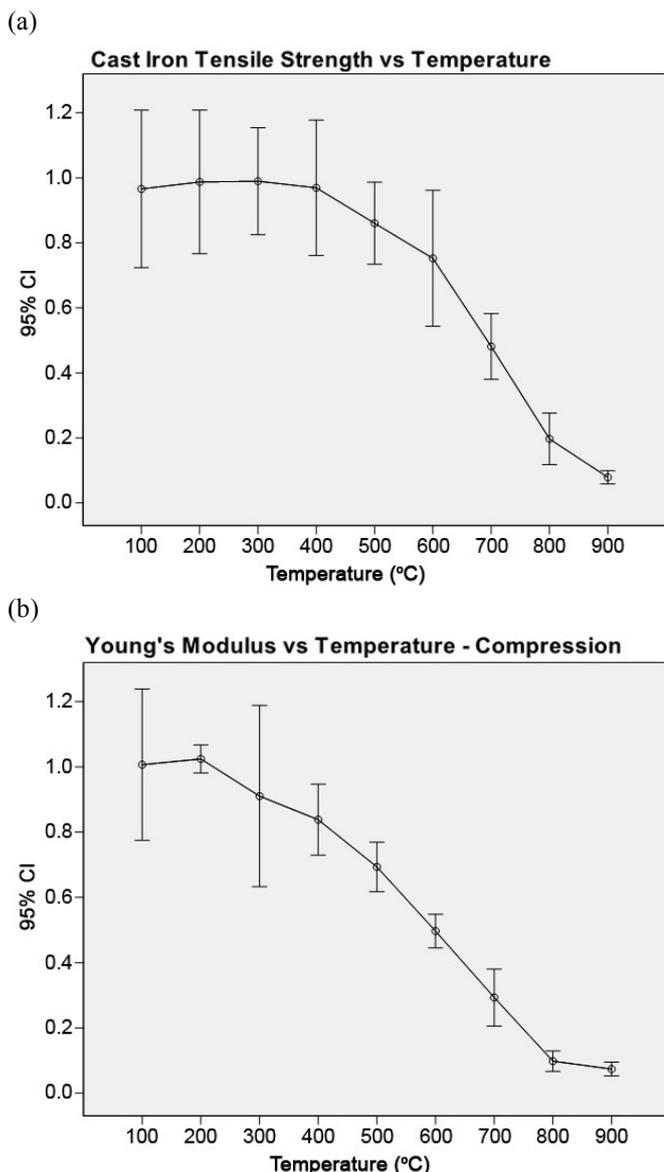


Figure 3. Typical diagrams of 95% confidence interval vs temperature for (a) tensile strength and (b) Young's modulus in compression.

## 6. TEMPERATURE PROFILES

The sections were assumed to be exposed to the standard fire and the thermal profiles of the cross-sections were calculated using the finite element software ABAQUS. Figure 5 shows the thermal boundary conditions and material properties used. The thermal properties of cast iron are those of steel according to EN1993-1-2 [17] and the thermal properties of the insulation are those of concrete according to EN1992-1-2 [23] as proposed by the authors in [5], [6], [7] and [8].

The temperature profiles of the sections are used as input in subsequent calculations of bending moment resistances of the cross-sections. This paper will present results for 30 and 60 minutes of

the standard fire exposure, being the most common fire ratings for such structures.

**Table 2. Mean and standard deviation for elevated temperature reduction factors in tension**

No	Stress-strain variable	Temperature (°C)	Mean	Standard Deviation
1	Young's modulus	100	1.1017	0.1949
2		200	1.0150	0.1088
3		300	1.0250	0.1484
4		400	0.9800	0.1170
5		500	0.8769	0.1239
6		600	0.6070	0.1520
7		700	0.3396	0.1353
8		800	0.1600	0.0762
9		900	0.1138	0.0254
10	0.2% proof stress	100	0.895	0.0495
11		200	0.9250	0.0636
12		300	0.9000	0.0282
13		400	0.9050	0.0353
14		500	0.8247	0.0487
15		600	0.5756	0.1908
16		700	0.3300	0.1199
17		800	0.1455	0.0493
18		900	0.0793	0.0178
19	Maximum stress	100	0.9658	0.1523
20		200	0.9873	0.1388
21		300	0.9896	0.1033
22		400	0.9693	0.1307
23		500	0.8687	0.0640
24		600	0.6351	0.1685
25		700	0.4891	0.1424
26		800	0.3026	0.1668
27		900	0.1195	0.0678
28	Strain at maximum stress	100	0.97611	0.339701
29		200	1.00597	0.356418
30		300	1.12835	0.310448
31		400	1.23880	0.437463
32		500	1.04850	0.620896
33		600	1.02238	0.536567
34		700	1.41791	0.581194
35		800	0.62194	0.430896
36		900	0.56223	0.161045

**Table 3. Mean and standard deviation for failure strain in tension**

No	Variable (strain (%))	Temperature (°C)	Mean	Standard Deviation
1	Failure strain	500	0.8275	0.5639
2		600	1.7429	0.7251
3		700	2.861	0.8466
4		800	3.7126	0.7079
5		900	5.6080	1.9608

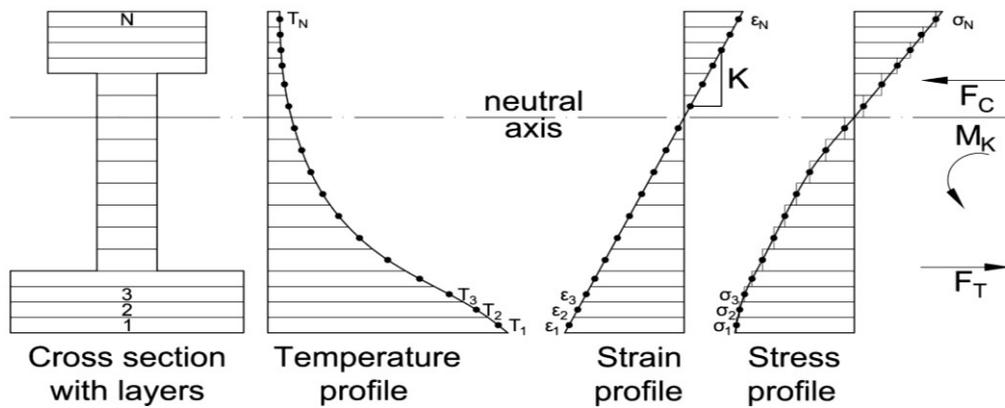


Figure 4. Schematic presentation of the fibre analysis procedure to obtain cast iron beam bending moment capacity [9]

**Table 4. Mean and standard deviation for elevated temperature reduction factors in compression**

No	Stress-strain variable	Temperature (°C)	Mean	Standard Deviation n
1	Young's modulus	100	0.9999	0.0698
2		200	1.0084	0.0249
3		300	0.9542	0.1015
4		400	0.8868	0.0739
5		500	0.6933	0.0306
6		600	0.4967	0.0208
7		700	0.2933	0.0351
8		800	0.0983	0.0125
9		900	0.0740	0.0085
10	Proportional limit	100	1.0003	0.0186
11		200	0.9934	0.0055
12		300	0.9855	0.0111
13		400	0.9652	0.0220
14		500	0.8220	0.0089
15		600	0.4033	0.0152
16		700	0.1461	0.0016
17		800	0.0589	0.0049
18		900	0.0337	0.0058
19	0.2% proof stress	100	0.9662	0.0449
20		200	0.9637	0.0398
21		300	0.9718	0.0344
22		400	0.9339	0.0483
23		500	0.6789	0.0222
24		600	0.3121	0.0398
25		700	0.1752	0.0037
26		800	0.0969	0.0081
27		900	0.0553	0.0092

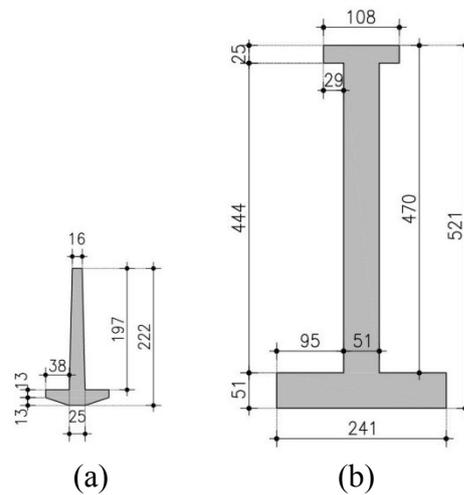


Figure 5: Cast iron cross section types used in the analysis, based on [21]: (a) Marshall mill (1817), jack arch span 3.35m, and (b) Shaw's H mill (1880), jack arch span 2.75m.

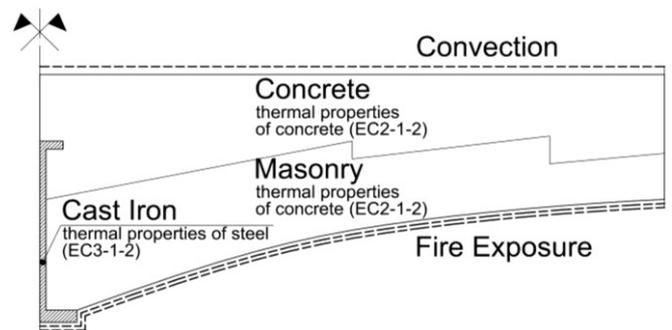


Figure 6. Thermal boundary conditions and thermal properties of materials used for the thermal analysis

## 7. METHODOLOGY OF RELIABILITY ANALYSIS

Monte-Carlo simulations were performed to estimate the probability of failure of the cast-iron beam cross-sections and the material safety factors. The material safety factor is calculated by the following equation:

$$\gamma_{M,fi} = \frac{M_{fi,T}}{M_{fi,T}^{P_f}} \quad (4)$$

where  $M_{fi,T}$  is the moment capacity calculated using the cast-iron mechanical property model in [8].  $M_{fi,T}^{P_f}$  is the moment capacity corresponding to the target probability of failure  $P_f$  at the fire exposure time  $T$ .

The probability of failure is defined as:

$$P_f = Prob[M_{fi,T} \leq M_{d,fi}] \quad (5)$$

where  $P_f$  is the probability of failure,  $M_{fi,T}$  the moment capacity at the fire exposure time  $T$  and  $M_{d,fi}$  the moment capacity at the same time  $T$  based on using the material model proposed in [8].

In the Monte Carlo simulations, the following nine elevated temperature mechanical properties of cast iron were varied:

- Young's modulus in tension
- The 0.2% proof stress in tension
- The maximum tensile stress
- The strain corresponding to the maximum tensile stress
- The strain at failure in tension
- Young's modulus in compression
- The proportional limit in compression
- The 0.2% proof stress in compression
- The maximum compressive stress

The mean and standard deviation values for these variables are given in Tables 2, 3 and 4.

The Monte Carlo simulation procedure is outlined below:

- For each Monte Carlo simulation, random values of the above nine variables at the corresponding temperatures were generated according to their distributions, assumed to be normal with the mean and

standard deviation values in Tables 2, 3 and 4. A total of 150,000 simulations were run, based on the rule of thumb that the sample size should exceed  $10/P_f$ , where the smallest  $P_f$  considered to be  $10^{-4}$ .

- Any negative property value was rejected.
- After selecting the nine random mechanical properties of cast iron, the stress-strain temperature relationships were generated.
- Use the elevated temperature stress-strain temperature relationships, for a given cross section and temperature profile, the moment resistance was calculated using the fibre analysis model outlined in section 4.
- From the calculated moment capacity results, the normal distribution parameters (mean, standard deviation) were calculated. Figure 7 shows typical results for Shaw's H cross section for 30 minutes of the standard fire exposure.
- From the calculated moment capacity distribution, the corresponding moment capacities for  $P_f = 10^{-1}$ ,  $10^{-2}$ ,  $10^{-3}$  and  $10^{-4}$  are calculated (equation (5)).

## 7. RESULTS

Tables 5 and 6 present the results of the reliability analysis.

From these results the short cross section (Figure 5a) needs higher material safety factor than the tall cross section (Figure 5b). This is expected as the temperature distribution affects a large part of the short cross section. In contrast, just a short part of the tall cross section experiences elevated temperatures.

The proposed material safety factors are high compared with the proposed values in Eurocodes for modern materials. This is expected, because the production and quality control of modern materials follow much more strict specifications than the cast iron beams manufactured during the 19th century when the production technology and quality control were at more primitive.

The safety factors for the higher fire rating, R60 are slightly higher than for the lower fire rating, R30. This is due to the larger scatter of tensile properties at higher temperatures associated with the higher fire rating. However, the differences in

the material safety factors for the two different fire ratings with the same probability of failure are relatively small. It is therefore possible to use the same material safety factor for different fire ratings. The safety factor to reach a failure probability of  $10^{-3}$ , being the likely lowest target probability to achieve a reliability index of 3.7, ranges from 4.19 to 5.53. This is very close to the ambient temperature safety factor of 5.0 [25]. The safety factors for the deeper Shaw's sections tend to be lower than those for the shallower Marshall cross-section. Again, this may be explained by the

higher temperatures, which are attained in the shallower Marshall cross-section. However, again the differences in the safety factors for the two beam sections are relatively small. To summarise, it is possible to recommend one set of material safety factors according to the target probability of failure, for different fire ratings and cross-section types. Approximately, the following safety factors may be used: 1.5, 2.5, 4.5 and 5.5 for target probabilities of  $10^{-1}$ ,  $10^{-2}$ ,  $10^{-3}$  and  $10^{-4}$  respectively.

**Table 5 Material safety factors for Shaw's H cross section (Figure 4b)**

Probability of failure $P_f$	Reliability index $\beta$	Moment Capacity(kNm) after standard fire exposure time of $M_{fi,T}^{P_f}$		Material safety factor $\gamma_{M,fi}$ for	
		30 minutes	60 minutes	30 minutes	60 minutes
$10^{-1}$	1.3	1176.35	779.40	1.33	1.44
$10^{-2}$	2.3	953.83	483.68	1.64	2.32
$10^{-3}$	3.1	791.13	267.47	1.98	4.19
$10^{-4}$	3.7	657.22	189.50	2.38	5.92
$M_{fi,LT}$ Material model [8]		1,565.73	1,122.39		

**Table 6 Material safety factors for Marshall's cross section (Figure 4a)**

Probability of failure $P_f$	Reliability index $\beta$	Moment Capacity(kNm) after standard fire exposure time of $M_{fi,T}^{P_f}$		Safety factor $\gamma_{M,fi}$ for	
		30 minutes	60 minutes	30 minutes	60 minutes
$10^{-1}$	1.3	58.64	28.02	1.52	1.58
$10^{-2}$	2.3	35.13	15.33	2.54	2.89
$10^{-3}$	3.1	24.66	8.01	3.62	5.53
$10^{-4}$	3.7	15.65	-	5.70	-
$M_{fi,LT}$ Material model [8]		89.34	44.37		

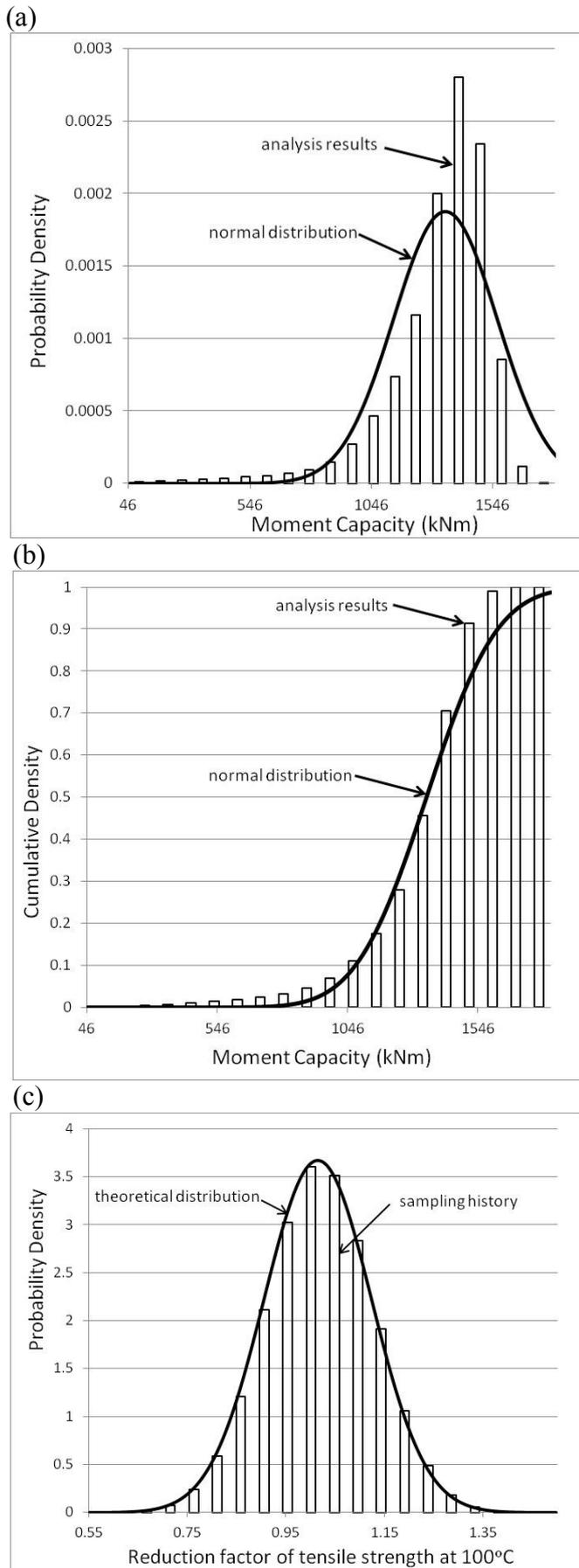


Figure 7 Typical probability distributions for mechanical property and bending moment capacity (for Shaw's H cross

section exposed for 30min) (a) Probability density of moment capacity (b) Cumulative probability of moment capacity and (c) Sampling history vs theoretical distribution of the reduction factor of tensile strength at 100°C.

## 8. CONCLUSIONS

This paper has presented the results of a Monte-Carlo simulation to derive material safety factors for cast iron beams. In this study, the beam bending moment capacity was calculated using a fibre analysis method. The mean and standard deviation values for the different key properties of the stress-strain-temperature relationships of cast iron in both tension and compression (Young's modulus in tension, the 0.2% proof stress in tension, the maximum tensile stress, the strain corresponding to the maximum tensile stress, the strain at failure in tension, Young's modulus in compression, the proportional limit in compression, the 0.2% proof stress in compression, the maximum compressive stress), were estimated from an assessment of a large amount of data collected by the authors, including the authors' own elevated temperature test data.

Based on an analysis of the probability of fire occurrence and the conditional probability of flashover given fire occurrence, the target failure probability given flashover was found to be in the range of  $1.0 \times 10^{-3}$  to achieve a reliability index of 3.8.

To achieve a target failure probability that is an order of magnitude smaller, the approximate safety factors are 1.50, 2.50, 4.50 and 5.50 for target probabilities of  $10^{-1}$ ,  $10^{-2}$ ,  $10^{-3}$  and  $10^{-4}$  respectively. The same material safety factors may be used for different cast iron beam cross-sections and different fire ratings.

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